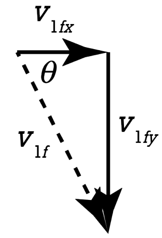
Flipping Physics Lecture Notes:

2D Conservation of Momentum Example using Air Hockey Discs

Example: A 28.8 g yellow air hockey disc elastically strikes a 26.9 g stationary red air hockey disc. If the velocity of the yellow disc before the collision is 33.6 cm/s in the x direction and after the collision it is 10.7 cm/s at an angle 63.4° S of E, what is the velocity of the red disc after the collision?

Knowns: $m_1 = 28.8g;\ m_2 = 26.9g;\ \vec{v}_{2i} = 0;\ \vec{v}_{1iy} = 0;\ \vec{v}_{1ix} = 33.6\frac{cm}{s};\ \vec{v}_{1f} = 10.7\frac{cm}{s}@63.4°S\ of\ E;\ \vec{v}_{2f} = ?$



Remember momentum is a vector so we need to break velocities into components in the x & y directions:

$$\cos\ \ = \frac{A}{H} = \frac{v_{1fx}}{v_{1f}} \Rightarrow v_{1fx} = v_{1f}\cos\ \ = (10.7)\cos(63.4) = 4.7910\frac{cm}{s}$$

$$\sin\theta = \frac{O}{H} = \frac{v_{1fy}}{v_{1f}} \Rightarrow v_{1fy} = v_{1f}\sin\theta = (10.7)\sin(63.4) = -9.5675\frac{cm}{s}$$ (negative because it is South)

Now we can use conservation of momentum in both the x and y directions:

x-direction: $\sum \vec{p}_{ix} = \sum \vec{p}_{fx} \Rightarrow m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx} \Rightarrow m_1v_{1ix} = m_1v_{1fx} + m_2v_{2fx}$

(because $v_{2ix} = 0$)

$$\Rightarrow m_1v_{1ix} - m_1v_{1fx} = m_2v_{2fx} \Rightarrow v_{2fx} = \frac{m_1v_{1ix} - m_1v_{1fx}}{m_2} = \frac{(28.8)(33.6) - (28.8)(4.7910)}{26.9}$$

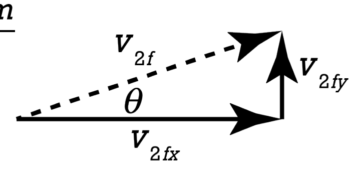
$$\Rightarrow v_{2fx} = 30.8438\frac{cm}{s}$$

note: $v_{2fx} \Rightarrow \frac{(g)\left(\frac{cm}{s}\right) - (g)\left(\frac{cm}{s}\right)}{g} \Rightarrow \frac{cm}{s}$

y-direction: $\sum \vec{p}_{iy} = \sum \vec{p}_{fy} \Rightarrow m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy} \Rightarrow 0 = m_1v_{1fy} + m_2v_{2fy}$

(because $v_{1iy} = v_{2iy} = 0$)

$$\Rightarrow -m_1v_{1fy} = m_2v_{2fy} \Rightarrow v_{2fy} = -\frac{m_1v_{1fy}}{m_2} = -\frac{(28.8)(-9.5675)}{26.9} = 10.2432\frac{cm}{s}$$



$$a^2 + b^2 = c^2 \Rightarrow {v_{2f}}^2 = {v_{2fx}}^2 + {v_{2fy}}^2 \Rightarrow v_{2f} = \sqrt{{v_{2fx}}^2 + {v_{2fy}}^2} = \sqrt{30.8438^2 + 10.2432^2} = 32.500 \approx 32.5\frac{cm}{s}$$

$$\tan\theta_{2f} = \frac{O}{A} = \frac{v_{2fy}}{v_{2fx}} \Rightarrow \theta_{2f} = \tan^{-1}\left(\frac{v_{2fy}}{v_{2fx}}\right) = \tan^{-1}\left(\frac{10.2432}{30.8438}\right) = 18.3713 \approx 18.4°$$

$\vec{v}_{2f} \approx 32.5\frac{cm}{s}@18.4°N\ of\ E$ (predicted)

Measured final velocity of the red air hockey disc is $\Rightarrow \vec{v}_{2f} \approx 32.0\frac{cm}{s}@13.0^\circ N\ of\ E$ (pretty close, eh!)

We consider this an elastic collision, so was kinetic energy conserved?
First, in order to work with energy, convert everything to base SI units!

$$m_1 = 28.8g \times \frac{1kg}{1000g} = 0.0288kg;\ m_2 = 26.9g \times \frac{1kg}{1000g} = 0.0269kg$$

$$v_{1i} = 33.6\frac{cm}{s} \times \frac{1m}{100cm} = 0.336\frac{m}{s};\ v_{1f} = 10.7\frac{cm}{s} \times \frac{1m}{100cm} = 0.107\frac{m}{s};\ v_{2f} = 32.5\frac{cm}{s} \times \frac{1m}{100cm} = 0.325\frac{m}{s}$$

$$\sum KE_i = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}(0.0288)(0.336)^2 = 0.0016257J$$ (because $v_{2i} = 0$)

$$\sum KE_i = 0.0016257J \times \frac{1000mJ}{1J} = 1.6257mJ$$

$$\sum KE_f = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}(0.0288)(0.107)^2 + \frac{1}{2}(0.0269)(0.320)^2 = 0.0015421J$$

$$\sum KE_f = 0.0015421J \times \frac{1000mJ}{1J} = 1.5421mJ$$

$$E_r = \frac{O-A}{A} \times 100 = \frac{1.5421 - 1.6257}{1.6257} \times 100 = -5.1397 \approx -5.14\%$$

In other words, 5.14% of the kinetic energy was converted to heat and sound during the “elastic” collision.