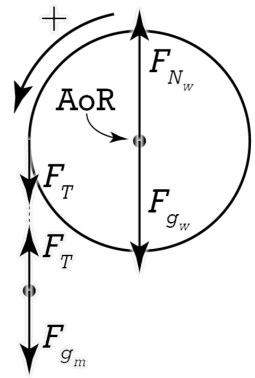
Flipping Physics Lecture Notes:

(1 of 2) Measuring the Rotational Inertia of a Bike Wheel



We are going to measure the rotational inertia (or moment of inertia) of a bike wheel. In order to do this we are going to attach a known mass to a string, wrap the string around the bike wheel, and let the mass apply a torque the bike wheel to angularly accelerate the wheel. We need to assume the axle of the bike wheel is frictionless.

FBD: F_N on bike wheel; F_g on bike wheel; F_g on hanging mass; F_T on bike wheel; F_T on hanging mass. r for both F_g on bike wheel and F_N on bike wheel are zero, so the torque caused by both of those forces is zero. Therefore, the only torque acting on the bike wheel is caused by the F_T of the hanging mass.

Sum the torques on only the wheel with the positive torque direction shown in the free body diagram:

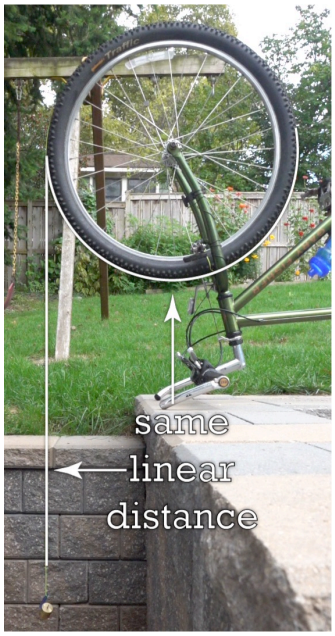
$$\sum \tau_{\substack{wheel \\ AoR@axle}} = +\tau_{F_T} = I\alpha \Rightarrow rF_T \sin\theta = RF_T \sin(90) = I\alpha \Rightarrow I = \frac{RF_T}{\alpha}$$

Sum the forces in the y-direction on just the hanging mass. Notice because of the way we have define positive torque that the positive y-direction is now down.

$$\sum F_{\substack{y \\ mass\,hanging}} = F_g - F_T = ma_y \Rightarrow mg - F_T = ma_y \Rightarrow F_T = mg - ma_y = m(g - a_y)$$

Which we can substitute back in to the rotational inertia equation:

$$\Rightarrow I = \frac{RF_T}{\alpha} = \frac{Rm(g - a_y)}{\alpha}$$



Because the string is connected to the hanging mass and the outside edge of the wheel, as the hanging mass goes down, the edge of the wheel travels the same linear distance or arc length. This means the acceleration in the y-direction of the hanging mass is the same as the tangential acceleration of the rim of the wheel, therefore:

$$a_y = a_t = r\alpha = R\alpha \Rightarrow I = \frac{Rm(g - R\alpha)}{\alpha}$$

Everything in the equation for the net force on the hanging mass is constant. This means the force of tension acting on the wheel is constant. In addition, the "r" vector of the hanging mass and the angle in the torque equation are all constant. This means the net torque acting on the wheel is constant. The rotational inertia of the wheel is also constant. Therefore, according to the rotational form of Newton's second law of motion, the angular acceleration of the wheel is constant and we can use the uniformly angularly accelerated motion equations.

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha\Delta t^2 = (0)\Delta t + \frac{1}{2}\alpha\Delta t^2 \Rightarrow 2\Delta\theta = \alpha\Delta t^2 \Rightarrow \alpha = \frac{2\Delta\theta}{\Delta t^2}$$

Therefore, to solve for the moment of inertia of the bike wheel, we need to know:

- Wheel radius
- Hanging mass
- Acceleration due to gravity of the planet we are on
- Change in time while the hanging mass accelerates downward
- Change in angular position of the bike wheel associated with the same change in time

Knowns: $r_{wheel} = 0.332m;\ m_{hanging} = 0.205kg;\ g_{Earth} = 9.81\frac{m^2}{s};\ \Delta t = 1.28\sec;$

$$\Delta\theta = 185^\circ \times \frac{2\pi rad}{360^\circ} = 3.228859rad\ \&\ \alpha = \frac{2\Delta\theta}{\Delta t^2} = \frac{(2)(3.228859)}{1.28^2} = 3.941478\frac{rad}{s^2}$$

$$I = \frac{Rm(g - R\alpha)}{\alpha} = \frac{(0.332)(0.205)(9.81 - (0.332)(3.941478))}{3.941478} = 0.146800 \approx \boxed{0.147kg \cdot m^2}$$