

Flipping Physics Lecture Notes: Capacitor Series and Parallel Circuits http://www.flippingphysics.com/capacitor-series-parallel.html

Let's look at two capacitors in parallel: We know the electric potential differences are all equal.

$$\Delta V_t = \Delta V_1 = \Delta V_2$$

Because the charges moved to the top plates of the capacitors need to go to either capacitor 1 or capacitor 2, the charge moved by the battery to the plates of the capacitors equals the sum of the charges on the capacitors:

$$Q_t = Q_1 + Q_2$$

We can then use the definition of capacitance:

$$C = \frac{Q}{\Lambda V} \Rightarrow Q = C \Delta V$$

To derive the equivalent capacitance of two capacitors in parallel:

 $\Rightarrow C_{\rm eq} \Delta V_t = C_1 \Delta V_1 + C_2 \Delta V_2 \Rightarrow C_{\rm eq} = C_1 + C_2$

And the equivalent capacitance of n capacitors in parallel:

$$\Rightarrow C_{\text{eq parallel}} = \sum_{n} C_{n} = C_{1} + C_{2} + \dots$$

$$\Delta V_t \frac{+}{-} \frac{+}{-} C_1 \frac{+}{-} C_2$$

And we can now look at two capacitors in series:

The electric potential is the same as resistors in series:

$$\Delta V_t = \Delta V_1 + \Delta V_2$$

And the charges on each capacitor are equal:

 $Q_t = Q_1 = Q_2$

This is because the magnitude of the charge moved by the battery to the top plate of capacitor 1 and the bottom plate of capacitor 2 are equal in magnitude. And those plates polarize the charges on the wire between the two capacitors and the bottom of capacitor 1 and the top of capacitor 2. This causes all four plates of the two capacitors to have equal magnitude charges. This is an illustration of conservation of charge.

And we can solve for electric potential difference in terms of capacitance and charge:

 $Q = C \Delta V \Rightarrow \Delta V = \frac{q}{C}$

And use that to solve for the equivalent capacitance of two capacitors:

$$\Rightarrow \frac{Q_t}{C_{eq}} = \frac{Q_1}{C_2} + \frac{Q_2}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

And the equivalent capacitance of n capacitors:

$$\Rightarrow C_{\text{eq series}} = \left(\sum_{n} \frac{1}{C_{n}}\right)^{-1} = \left(\frac{1}{C_{1}} + \frac{1}{C_{2}} + \dots\right)^{-1}$$

Notice the equations for resistors and capacitors are reversed. That means that: When we add a capacitor in parallel, the equivalent capacitance increases. When we add a capacitor in series, the equivalent capacitance decreases.