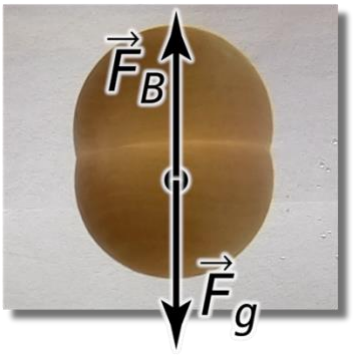
Flipping Physics Lecture Notes:
Buoyant Force Explained: Objects Floating on Fluids!
http://www.flippingphysics.com/buoyant-force-float.html

We've talked a lot about the buoyant force acting on objects submerged in fluids. Today we are going to look at the buoyant force acting on an object floating in a fluid.

Example: A wooden sphere floats on water. Determine the percentage of the volume of the wood sphere which is below the surface of the water. The density of water is 1.00 x 10^3 kg/m^3. The density of this wood is 660 kg/m^3.



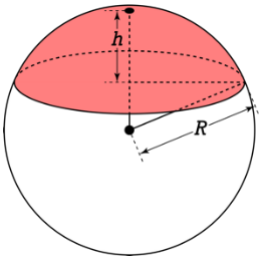
Knowns: $\rho_{water} = \rho_f = 1.00\times10^3 \frac{kg}{m^3}; \rho_{wood} = \rho_o = 660\frac{kg}{m^3}; \frac{V_f}{V_o} = ?\,(\%)$

$$\sum F_y = F_B - F_g = m_o a_y = m_o(0) = 0 \Rightarrow F_B = F_g$$

$$\Rightarrow m_f g = m_o g \Rightarrow m_f = m_o \;\&\; \rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow \rho_f V_f = \rho_o V_o$$

$$\&\text{ not submerged} \rightarrow V_f \neq V_o \Rightarrow \frac{V_f}{V_o} = \frac{\rho_o}{\rho_f} = \frac{660}{1000} = 0.66$$

So, we have determined that 66% of the wooden sphere is below the surface of the water. Incidentally, that means 34% of the wooden sphere is above the surface of the water.



In order to test this, let's use an equation from Wolfram MathWorld.[1]

The equation is for a spherical cap: $V_{cap} = \frac{1}{3}\pi h^2(3R - h)$

Knowns: $D = 50.7mm\left(\frac{1m}{1000mm}\right) = 0.0507m \Rightarrow R = \frac{D}{2} = \frac{0.0507}{2} = 0.02535m$

$$h = 2.0cm\left(\frac{1m}{100cm}\right) = 0.02m$$

$$V_o = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.02535)^3 = 6.82374\times10^{-5}m^3$$

$$\Rightarrow V_{cap} = \frac{1}{3}\pi(0.02)^2((3)(0.02535) - 0.02) = 2.34782\times10^{-5}m^3$$

$$\frac{V_{cap}}{V_o} = \frac{2.34782\times10^{-5}}{6.82374\times10^{-5}} = 0.3441 \approx 0.34 \Rightarrow 34\%$$

We just showed that 34% of the wood sphere is above the surface of the water. That matches our prediction. The physics works!

[1] Wolfram MathWorld – Spherical Cap