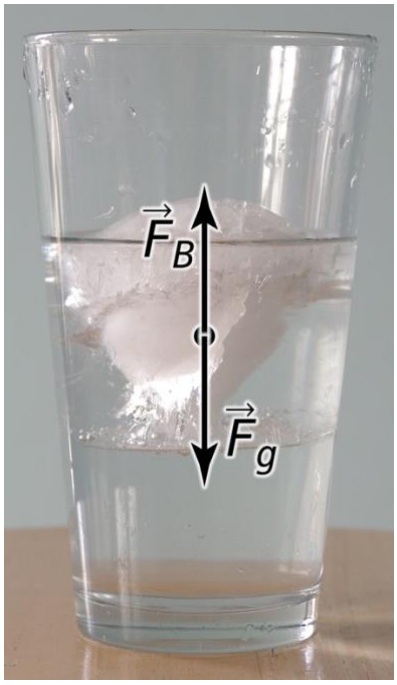
Flipping Physics Lecture Notes:
Ice Melting in Water: Does the Water Level Change?
http://www.flippingphysics.com/buoyant-force-ice.html

We've talked a lot about the buoyant force. Today we are going to look at a classic buoyancy question.



Example: A chunk of ice floats in a glass of water. As the ice melts, does the water level in the glass go up, down, or remain the same?

Start with the free body diagram of the forces acting on the chunk of ice. The buoyant force is up and the force of gravity is down. And then we sum the forces:

$$\sum F_y = F_B - F_g = m_o a_y = m_o(0) = 0$$
$$\Rightarrow F_B = F_g \Rightarrow m_f g = m_o g \Rightarrow m_f = m_o$$

In other words, the mass of the fluid displaced by the ice is the same as the mass of the ice. And we know the equation for density:

$$\rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow \rho_f V_f = m_o$$

In other words, if we were to remove the chunk of ice and replace it with water, the volume of the water we would need to use would equal the empty space below the waterline where the chunk of ice used to be. As the ice melts, that's exactly what happens and the water level remains the same.