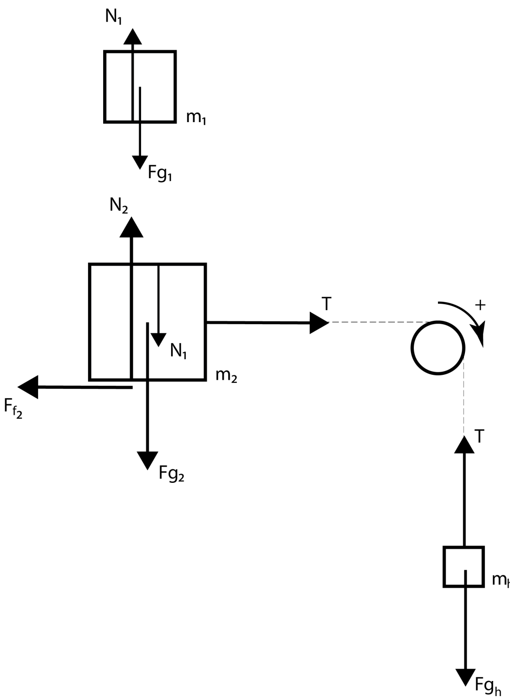
Flipping Physics Lecture Notes:
Mechanics Free Response Question #3 Solutions
AP Physics C 1998 Released Exam from the College Board

For legibility, please observe this legend: $f_1 = F_{f_1}, f_2 = F_{f_2}, \& \; M = m_h\,(for\ mass\ hanging)$

Knowns, m_1, m_2, m_h, μ_{s1}, μ_{k1}, μ_{s2}, μ_{k2}. Give answers in terms of these variables and g.



Complete Free Body Diagram:

(a) v = 0

(a i) The normal force N_1 exerted on block 1 by block 2 is up and acts on the bottom surface of m_1.

$$\sum F_{y(on\,m_1)} = N_1 - F_{g1} = m_1 a_y = m_1(0) = 0 \Rightarrow \boxed{N_1 = F_{g1} = m_1 g}$$

(a ii) Because m_1 is not moving, there is no friction force F_{f1} exerted on block 1 by block 2. $\boxed{F_{f1} = 0}$

(a iii) The Tension force *T* exerted by a string is always a pull in the direction of the string. Therefore *T* is to the right and acts right where the string attaches to block 2. Let's set to the right on m_2 and down on m_h as the positive direction.

$$\sum F_{y(on\,m_h)} = -T + F_{gh} = m_h a_y = m_h(0) = 0 \Rightarrow \boxed{T = F_{gh} = m_h g}$$

(a iv) The normal force N_2 exerted on block 2 by the tabletop is up and acts on the bottom surface of m_2.

$$\sum F_{y(on\,m_2)} = N_2 - F_{g2} - N_1 = m_2 a_{2y} = m_2(0) = 0$$

$$\Rightarrow N_2 = F_{g2} + N_1 = m_2 g + m_1 g \Rightarrow \boxed{N_2 = (m_2 + m_1)g}$$

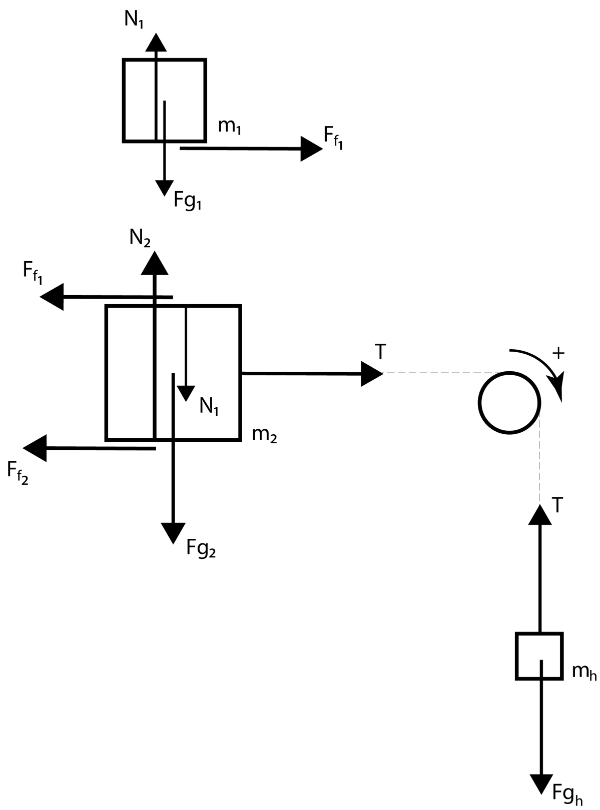
(a v) The friction force F_{f2} exerted on block 2 by the tabletop is keeping the block from moving and is therefore to the left and acts on the bottom surface of m_2.

$$\sum F_{x(on\,m_2)} = T - F_{sf_2} = m_2 a_{2x} = m_2(0) = 0 \Rightarrow \boxed{F_{sf_2} = T = m_h g}$$

(b) Sum the forces on all the objects in the positive direction indicated on the pulley in the diagram.

$$\sum F_{\text{all}\ in+dir} = -F_{sf_2} + T - T + F_{g_h} = m_t a = m_t(0) = 0 \Rightarrow F_{sf_2} = F_{g_h}$$

$$\Rightarrow \mu_{s2} N_2 = m_h g \Rightarrow \mu_{s2}(m_1 g + m_2 g) = m_h g \Rightarrow \boxed{m_h = \mu_{s2}(m_1 + m_2)}$$



(c) The blocks are now accelerating, however, they are not slipping relative to one another and therefore have the same acceleration in the x direction. The only change in the Free Body Diagram is the addition of **two** friction forces F_{f1} between block 1 and block 2 (because they are a Newton's 3rd Law interaction pair). F_{f1} on block 2 is to the left and acts on the top of block 2. F_{f1} on block 1 is to the right and acts on the bottom of block 1.

Because there are no additional forces in the y-direction on either block, both Newton's 2nd law equations in the y-direction are still valid. Therefore $N_1 = m_1 g$ and $N_2 = m_1 g + m_2 g$.

Now we sum the forces on the *whole system* in the same direction we had defined as positive before, which is to the right on block 2 and down on the mass hanging.

$$\sum F_{x\,system} = F_{sf_1} - F_{sf_1} + T - F_{kf_2} - T + F_{gM} = m_{total}a_{total} \Rightarrow -\mu_{k2}N_2 + m_h g = \left(m_1 + m_2 + m_h\right)a$$

$$\Rightarrow a = \frac{-\mu_{k2}\left(m_1 g + m_2 g\right) + m_h g}{m_1 + m_2 + m_h} = \boxed{\frac{m_h - \mu_{k2}\left(m_1 + m_2\right)}{m_1 + m_2 + m_h}\left(g\right)}$$

(d) Now the two blocks are slipping relative to one another; this changes F_{f1} to kinetic friction. Again there are no additional forces in the y-direction on either block, both Newton's 2nd law equations in the y-direction are still valid. Therefore $N_1 = m_1 g$ and $N_2 = m_1 g + m_2 g$.

(d i) $\sum F_{x m_1} = F_{kf_1} = m_1 a_{1x} \Rightarrow \mu_{k1} N_1 = \mu_{k1} m_1 g = m_1 a_{1x} \Rightarrow \boxed{a_{1x} = \mu_{k1} g}$

(d ii) Now we sum the forces on m_2 and m_h again in the "positive direction".

$$\sum F_{m_2\,\&\,M+dir} = -F_{kf_1} - F_{kf_2} + T - T + F_{gh} = \left(m_2 + m_h\right)a_{m_2\,\&\,m_h} \Rightarrow -\mu_{k1}N_1 - \mu_{k2}N_2 + m_h g = \left(m_2 + m_h\right)a$$

$$\Rightarrow -\mu_{k1}m_1 g - \mu_{k2}\left(m_1 g + m_2 g\right) + m_h g = \left(m_2 + m_h\right)a \Rightarrow a = \boxed{\frac{\left(g\right)\left(m_h - \mu_{k1}m_1 - \mu_{k2}\left(m_1 + m_2\right)\right)}{\left(m_2 + m_h\right)}}$$

Notes about the published solutions:

(a) Be careful to indicate *where* the forces act on the objects. Students often forget and have them all acting on the center of mass of the object.

- This whole problem is really just drawing Free Body Diagrams and using Newton's 2nd law repeatedly. This should indicate to you the importance of those very basic skills.