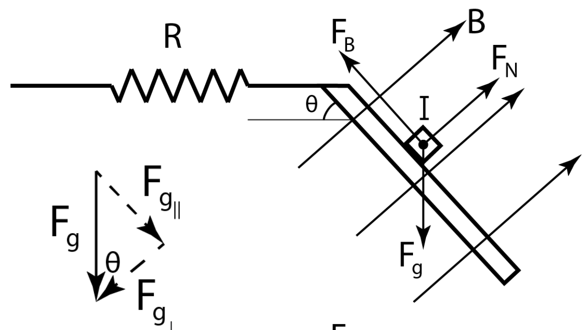
Friction is negligible and express all of our answers for (a) – (d) in terms of m, ℓ, θ, B, R and g.

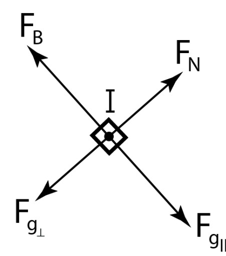
Part (a): The bar is has reached a constant, terminal velocity. The positively charged particles in the bar are moving with a constant velocity, v, perpendicular to the magnetic field. Using the right hand rule, point your fingers in the direction of the velocity or down the rails, curl your fingers in the direction of the magnetic field or up and normal to the rails, your thumb points in the direction of the magnetic force on the positively charged particle which is toward the front rail. This is the direction of the current in the rail which means there is a magnetic force on the rail due to the current in the rail. Point your fingers in the direction of the current or along the bar toward the front rail, curl your fingers in the direction of the magnetic field or up and perpendicular to the rails and your thumb points up the rails and in the direction of the magnetic force on the bar which is up the rails. The equation for the magnetic force on the current carrying bar is:

$F_B = IlB\sin\theta = IlB\sin(90) = IlB$ (the current and the magnetic field are normal to one another). The free body diagram in a side view is to the right (the current is toward you in the picture). In order to solve for the current, we need to sum the forces. In order to sum the forces, we need to break forces in to components. Because we are going to sum the forces parallel to the rails, we don't break the magnetic force into components, rather we break the force of gravity into components.



$$\sin\theta = \frac{O}{H} = \frac{F_{g_{\parallel}}}{F_g} \Rightarrow F_{g_{\parallel}} = F_g\sin\theta = mg\sin\theta$$

$$\cos\theta = \frac{A}{H} = \frac{F_{g_{\perp}}}{F_g} \Rightarrow F_{g_{\perp}} = F_g\cos\theta = mg\cos\theta$$ & Redraw the Free Body Diagram



Now we can sum the forces in the parallel direction:

$$\sum F_{\parallel} = F_B - F_{g_{\parallel}} = ma_{\parallel} = m(0) \Rightarrow F_B = F_{g_{\parallel}} \Rightarrow IlB = mg\sin\theta \Rightarrow \boxed{I = \frac{mg\sin\theta}{lB}}$$

Part (b): In order to find the constant speed of the bar, we need to first find the motional emf across the bar.

$$\varepsilon = -\frac{d\phi_B}{dt} \Rightarrow \|\varepsilon\| = -\frac{d}{dt}(BA\cos\theta) = -B\cos(180)\frac{d}{dt}(xl) = Bl\frac{dx}{dt} = Blv$$ Note: the angle between the area vector and the magnetic field is 180°, however, we only need the magnitude here. Also, the area is xℓ, where x is the variable distance from the top of the rail to where the bar touches the rail. Now we can solve for the velocity:

$$\varepsilon = \Delta V = IR \Rightarrow Blv = IR = \left(\frac{mg\sin\theta}{lB}\right)(R) \Rightarrow v = \boxed{\frac{mgR\sin\theta}{l^2B^2}}$$

Part (c): $P = I^2R = \left(\frac{mg\sin\theta}{lB}\right)^2 R = \boxed{\frac{m^2g^2R\sin^2\theta}{l^2B^2}}$

Part (d): We need to find the speed of the bar as a function of time. The math will be similar to charging a capacitor through a resistor (an RC circuit). We know the limits. $v_i = 0$ & $v_f = \frac{mgR\sin\theta}{l^2B^2}$ Here we go.

Let's use the same free body diagram, however, the acceleration in the parallel direction is no longer zero and let's define down the rails as positive so that our acceleration will be positive.

$$\sum F_{ll} = -F_B + F_{g_{ll}} = ma_{ll} \Rightarrow -ILB + mg\sin\theta = m\frac{dv}{dt} \Rightarrow \frac{dv}{dt} = g\sin\theta - \frac{IlB}{m}$$

& $\|\varepsilon\| = Blv = \Delta V = IR \Rightarrow I = \frac{Blv}{R}$ therefore: $\frac{dv}{dt} = g\sin\theta - \left(\frac{Blv}{R}\right)\left(\frac{lB}{m}\right) = g\sin\theta - \frac{l^2B^2v}{mR}$

We can now rearrange and take the integral of both sides.

$$\frac{dv}{g\sin\theta - \frac{l^2B^2v}{mR}} = dt \Rightarrow \int_{v_i}^{v_f} \frac{dv}{g\sin\theta - \frac{l^2B^2v}{mR}} = \int_{t_i}^{t_f} dt = t_f - t_i = t - 0 = t$$

I solved the right hand side of the equation first, now let's solve the left hand side (and switch sides).

$$\Rightarrow t = \frac{\ln\left[g\sin\theta - \frac{B^2l^2v}{MR}\right]_{v_i=0}^{v_f=v}}{-\frac{B^2l^2}{MR}} \Rightarrow -\frac{B^2l^2t}{MR} = \left[\ln\left(g\sin\theta - \frac{B^2l^2v}{MR}\right) - \ln\left(g\sin\theta - \frac{B^2l^2(0)}{MR}\right)\right]$$

$$\Rightarrow -\frac{B^2l^2t}{MR} = \ln\left(\frac{g\sin\theta - \frac{B^2l^2v}{MR}}{g\sin\theta}\right) \Rightarrow e^{\ln\left(\frac{g\sin\theta - \frac{B^2l^2v}{MR}}{g\sin\theta}\right)} = e^{-\frac{B^2l^2t}{MR}} \Rightarrow \frac{g\sin\theta - \frac{B^2l^2v}{MR}}{g\sin\theta} = e^{-\frac{B^2l^2t}{MR}}$$

$$\Rightarrow g\sin\theta - \frac{B^2l^2v}{MR} = (g\sin\theta)e^{-\frac{B^2l^2t}{MR}} \Rightarrow -\frac{B^2l^2v}{MR} = (g\sin\theta)e^{-\frac{B^2l^2t}{MR}} - g\sin\theta \Rightarrow \frac{B^2l^2v}{MR} = g\sin\theta\left(1 - e^{-\frac{B^2l^2t}{MR}}\right)$$

$$\Rightarrow \boxed{v = \frac{MRg\sin\theta}{B^2l^2}\left(1 - e^{-\frac{B^2l^2t}{MR}}\right)}$$

That was a bit complicated; so let's check our limits. We know at t = 0 that v = 0.

$$v(0) = \frac{MRg\sin\theta}{B^2l^2}\left(1 - e^{-\frac{B^2l^2(0)}{MR}}\right) = \frac{MRg\sin\theta}{B^2l^2}(1-1) = 0 \qquad \text{(that is correct)}$$

We also know that at time $t \approx \infty$ that $v = \frac{mgR\sin\theta}{l^2B^2}$

$$v(\infty) = \frac{MRg\sin\theta}{B^2l^2}\left(1 - e^{-\frac{B^2l^2(\infty)}{MR}}\right) = \frac{MRg\sin\theta}{B^2l^2}(1-0) = \frac{MRg\sin\theta}{B^2l^2} \qquad \text{(glad that worked too)}$$

Part (e):

The velocity final is directly proportional to the resistance of the circuit: $v_f = \frac{mgR\sin\theta}{l^2B^2}$

When you add the resistor to the circuit at the bottom of the rails you are adding a resistor in parallel which will decrease the resistance of the circuit and therefore the velocity final will decrease.