Flipping Physics Lecture Notes:

Work-Energy Theorem Problem
by Billy

This video is a second alternate solution to a problem previously done by Billy.
http://www.flippingphysics.com/coe-incline-problem.html

Here is the first alternate solution:
http://www.flippingphysics.com/wf-problem-billy.html

Example: A block with a mass of 11 grams is used to compress a spring a distance of 3.2 cm. The spring constant of the spring is 14 N/m. After the block is released, it slides along a level, frictionless surface until it comes to the bottom of a 25° incline. If μk between the block and the incline is 0.30, to what maximum height does the block slide?

We have solved this problem using 1) Conservation of Energy, Newton's Second Law and Uniformly Accelerated Motion and 2) $W_f = \Delta ME$. Now we are going to solve the problem using the Work-Energy Theorem or what Mr. P prefers to call the Net Work-Kinetic Energy Theorem, because, evidently we often confuse $W_{net} = \Delta KE$ with $W_f = \Delta ME$.

We have the same knowns as the last two times we solved this problem:

$$k = 14\frac{N}{m};\ \theta = 25°;\ m = 11g;\ x_i = 3.2cm;\ \mu_k = 0.30;\ h_{max} = ?$$

$$m = 11g \times \frac{1kg}{1000g} = 0.011kg \ \ \&\ \ x_i = 3.2cm \times \frac{1m}{100cm} = 0.032m$$

And we now begin with $W_{net} = \Delta KE = KE_f - KE_i$ and we need to identify the locations of the initial and final points. Set the initial point where spring is compressed its maximum distance and the final point at the maximum height of the block. **PICTURE** Because the velocity initial and final are both zero, there is no Kinetic Energy initial or final, in other words:

$W_{net} = KE_f - KE_i = 0 - 0 = 0 \Rightarrow W_{net} = 0$ The net work done on the block is zero.

The equation for work is: $W = Fd\cos\theta$

Because the direction the block is moving is different on the horizontal, frictionless surface than on the incline, let's separate the net work into those two parts:

Horizontal Surface:
We need to draw the **free body diagram** for the forces acting on the block when it is on the horizontal surface. The angle between the direction the block is moving (to the right) and the Force Normal (up) is 90°. The cosine of 90° is zero, so the work done by the Force normal is zero. The same is true for the Force of Gravity. $W_{F_N} = F_N d\cos(90) = 0 = F_g d\cos(90) = W_{F_g}$

Because the force caused by the spring on the block is not constant, we need to find the work done by the spring on the block a little bit differently. As the block is pushed by the spring, the spring loses Elastic Potential Energy, that loss in Elastic Potential Energy equals the Work done by the Force of the Spring on the block. In equation form that is:

$$W_{F_s} = -\Delta PE_e = -\left(PE_{e_f} - PE_{e_i}\right) = -\left(0 - PE_{e_i}\right) = PE_{e_i} = \frac{1}{2}kx_i^2$$

Because the block is not on the spring finally, the final Elastic Potential Energy of the spring is zero. Therefore the Work done by the Spring on the block equals the Elastic Potential Energy initial of the spring.

Therefore the Net Work done on the block on the horizontal surface is: $W_{F_s} = \frac{1}{2}kx_i^2$

Incline:
Again, we need the **free body diagram** on the incline. Again the work done by the Force Normal is zero because the Force Normal is perpendicular to the displacement of the block.

$$W_{F_N} = F_N \Delta d_{\parallel} \cos\left(90\right) = 0$$

The work done by the Force of Gravity is: $W_{F_g} = Fd\cos\theta = F_g \Delta d_{\parallel} \cos\theta_1$

The equation for the Force of Gravity is: $F_g = mg$

The displacement of the block on the incline is the displacement in the parallel direction.

There are going to be several angles in this problem, so I have labeled this angle θ_1. This is the angle between the displacement of the block (up the incline) and the Force of Gravity (down), which is 90° + 25° or 115°. 1 minute and 29 seconds in to this previous video, I showed finding this angle in detail: http://www.flippingphysics.com/work-billy.html

We now have: $W_{F_g} = mg\Delta d_{\parallel} \cos\theta_1$ with $\theta_1 = 115°$

We need $\Delta d_{\parallel}$ in terms of h_{max}, so we draw a **triangle** and $\sin\theta_2 = \frac{O}{H} = \frac{h_{max}}{\Delta d_{\parallel}} \Rightarrow \Delta d_{\parallel} = \frac{h_{max}}{\sin\theta_2}$

Therefore: $W_{F_g} = mg\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_1 = \frac{mgh_{max}\cos\theta_1}{\sin\theta_2}$ where $\theta_2 = 25°$ (the incline angle)

The work done by the Force of Kinetic Friction is:

$$W_{F_{kf}} = Fd\cos\theta = F_{kf}\Delta d_{\parallel}\cos\theta_3 = \mu_k F_N\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_3$$

Now we need the work done by the Force of Kinetic Friction.

Remember the equation for the Force of Kinetic Friction is $F_{kf} = \mu_k F_N$

θ_3 is the angle between the direction of the displacement of the block (up the incline) and the force of kinetic friction (down the incline) which is 180°. $\theta_3 = 180°$

We need Force Normal so we need to break the force of gravity into its components, redraw the **free body diagram** and sum the forces in the perpendicular direction:

$$\sum F_\perp = F_N - F_{g_\perp} = ma_\perp = m(0) = 0 \Rightarrow F_N = F_g = mg\cos\theta_2$$

Which we can substitute back into the equation for the Work done by Friction:

$$W_{F_{kf}} = \mu_k\left(mg\cos\theta_2\right)\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_3 = \frac{\mu_k mgh_{max}\cos\theta_3}{\tan\theta_2}$$ (because $\frac{\cos\theta_2}{\sin\theta_2} = \frac{1}{\tan\theta_2}$)

Going back to the original Net Work-Kinetic Energy Theorem:

$$W_{net} = \Delta KE = 0 = W_{F_s} + W_{F_g} + W_{f_{kf}}$$

Now we can substitute in equations and numbers and solve for h_{max}:

$$\Rightarrow W_{net} = 0 = \frac{1}{2}kx_i^{\,2} + \frac{mgh_{max}\cos\theta_1}{\sin\theta_2} + \frac{\mu_k mgh_{max}\cos\theta_3}{\tan\theta_2}$$

$$\Rightarrow 0 = \frac{1}{2}(14)(0.032)^2 + \frac{(0.011)(9.81)h_{max}\cos(115)}{\sin(25)} + \frac{(0.30)(0.011)(9.81)h_{max}\cos(180)}{\tan(25)}$$

$$\Rightarrow 0 = 0.007168 + (-0.10791)h_{max} + (-0.069424)h_{max} \Rightarrow 0.177334h_{max} = 0.007168$$

$$h_{max} = 0.040421 \approx \boxed{0.040m}$$

(yes, this is the same answer we got the previous two times we solved this problem.)

It is important to note that:
The net work put into the system by the Force Applied is the same as the work done by the Force of the Spring which is positive because the spring converts Elastic Potential Energy to Kinetic Energy. $\sum W_{in} = PE_e = W_{F_s} = \frac{1}{2}kx_i^{\,2} = \frac{1}{2}(14)(0.032)^2 = 0.007168J$

The work done by the Force of Kinetic Friction is negative because it converts Kinetic Energy into heat and sound energy.

$$W_{F_{kf}} = (-0.069424)h_{max} = (-0.069424)(0.040421) = -0.0028062J$$

The work done by the Force of Gravity is negative because it converts the Kinetic Energy into Gravitational Potential Energy.

$$W_{F_g} = (-0.10791)h_{\max} = (-0.10791)(0.040421) = -0.0043618J$$

Because: $W_{F_g} = \dfrac{mgh_{\max}\cos\theta_1}{\sin\theta_2} = mgh_{\max}\left(\dfrac{\cos(115)}{\sin(25)}\right) = mgh_{\max}(-1) = -mgh_{\max}$

Which means $\dfrac{W_{F_{kf}}}{\sum W_{in}} \times 100 = \dfrac{-0.0028062}{0.007168} \times 100 = -39.149 \approx -39\%$ of the total energy put into the system by the force applied was dissipated as heat and sound energy.

And $\dfrac{W_{F_g}}{\sum W_{in}} \times 100 = \dfrac{-0.0043618}{0.007168} \times 100 = -0.60851 \approx -61\%$ of the total energy put into the system by the force applied was converted to Gravitational Potential Energy.