Flipping Physics Lecture Notes:

Solving for Force of Drag on an Accelerating Car

In my previous video "Average Power Delivered by a Car Engine" there was an "average drag force of 8.0 N" acting on a Prius. Here is how I solved for that number. Previous Video: http://www.flippingphysics.com/average-power.html

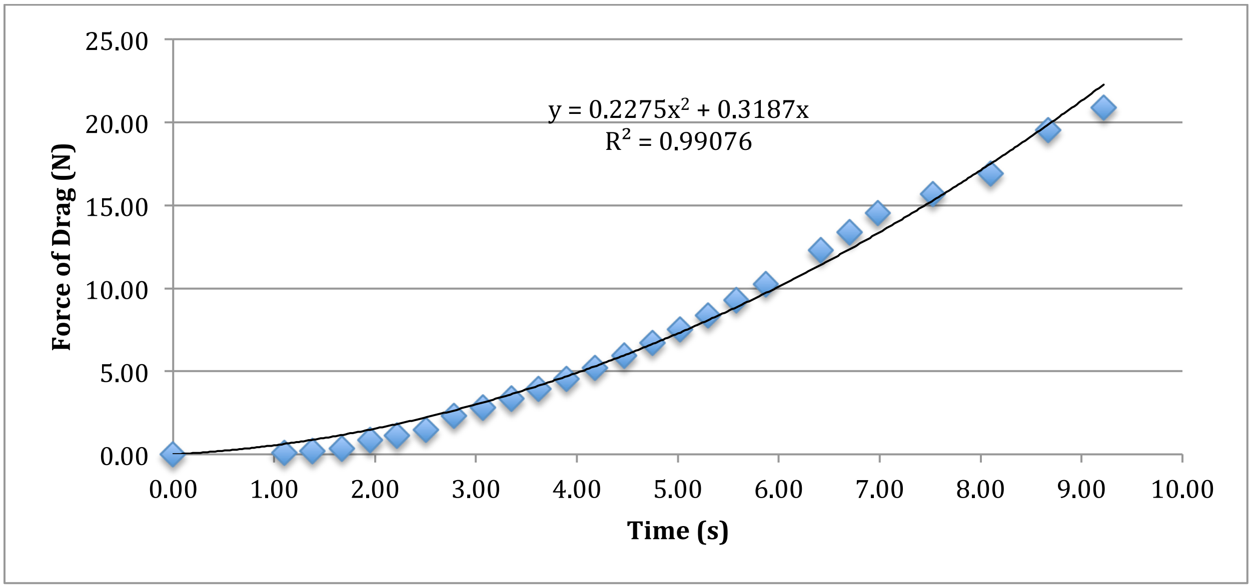
A standard equation for the Force of Drag is $F_{Drag} = \frac{1}{2}\rho v^2 DA$.

- ρ, is the density of the medium through which the object is moving. In our example, the density of air. In order to know the density of air, we need the air temperature, which was 67 °F.
  - $Temp_{air} = 67°F \Rightarrow T_{°C} = \left(T_{°F} - 32\right)\left(\frac{5}{9}\right) = \left(67 - 32\right)\left(\frac{5}{9}\right) = 19.\overline{4} \approx 19°C$
  - According to The Engineering Toolbox[1], 1.2 kg/m^3 is a good approximation for the density of air at this temperature.
- D, is the Drag Coefficient of the object, in this case the Prius. According to EcoModder[2], the Drag Coefficient of my 2011 Toyota Prius is 0.25.
- A, is the Cross Sectional Area perpendicular to the direction of motion. This is the Frontal Area listed on the same EcoModder page. It is 21.6 square feet. Of course, we need it in square meters.
  - $A = 21.6ft^2 \times \left(\frac{1m}{3.281ft}\right)^2 = 2.00651m^2$
- v, is the velocity of the car. Unfortunately, because the velocity is squared in the equation for the Force of Drag, we cannot simply find the average velocity and use that. Instead, we need to use the instantaneous velocity of the car to plot the instantaneous force of drag as a function of time and use an integral.

Time (s)	Interval Time (s)	Instantaneous Speed (km/hr)	Instantaneous Speed (m/s)	Instantaneous Force of Drag (N)
0.00		0	0.00	0.00
1.10	1.10	2	0.56	0.093
1.38	0.28	3	0.83	0.21
1.67	0.29	4	1.11	0.37
1.95	0.28	6	1.67	0.84
2.22	0.27	7	1.94	1.14
2.50	0.28	8	2.22	1.49
2.78	0.28	10	2.78	2.32
3.07	0.29	11	3.06	2.81
3.35	0.28	12	3.33	3.34
3.62	0.27	13	3.61	3.92
3.90	0.28	14	3.89	4.55
4.18	0.28	15	4.17	5.23
4.47	0.29	16	4.44	5.95
4.75	0.28	17	4.72	6.71
5.02	0.27	18	5.00	7.52
5.30	0.28	19	5.28	8.38

[1] http://www.engineeringtoolbox.com/air-density-specific-weight-d_600.html
[2] http://ecomodder.com/wiki/index.php/Vehicle_Coefficient_of_Drag_List

Time (s)	Interval Time (s)	Instantaneous Speed (km/hr)	Instantaneous Speed (m/s)	Instantaneous Force of Drag (N)
5.58	0.28	20	5.56	9.29
5.87	0.29	21	5.83	10.24
6.42	0.55	23	6.39	12.29
6.70	0.28	24	6.67	13.38
6.98	0.28	25	6.94	14.51
7.53	0.55	26	7.22	15.70
8.10	0.57	27	7.50	16.93
8.67	0.57	29	8.06	19.53
9.22	0.55	30	8.33	20.90



Here is what I did to create the table and graph above:

- Use the video to determine the time when the speedometer reading changed. This is the “Time” column.
- Determine the interval for each change in the speedometer reading. This is the “Interval Time” column.
  - Notice it appears the Prius’ speedometer updates slightly more than three times every second.
  - The “Instantaneous Speed” is the speedometer reading. Originally this is in kilometers per hour.
- Convert “Instantaneous Speed” to meters per second.
  - Last reading: $Instantaneous\ Speed = 30\frac{km}{hr}\times\frac{1hr}{3600\,\text{sec}}\times\frac{1000m}{1km} = 8.\overline{3}\frac{m}{s}$
- Determine the “Instantaneous Force of Drag”. This is the force of drag at each time and it is “instantaneous” because it uses the velocity at that specific point in time.
  - Last reading: $F_{Drag} = \frac{1}{2}\rho v^2 DA = \frac{1}{2}(1.2)(8.\overline{3})^2(0.25)(2.00651) = 20.90N$
- Plot all of the data: Force of Drag as a function of Time. Note the “R squared value” of 0.999 is very close to 1. A value of 1 would be a 100% perfect fit, our best-fit curve is 99.9% accurate, which is quite good.

- Add a best-fit curve. Excel reports $y = 0.2275x^2 + 0.3187x$ as the best-fit curve. However, we know Force of Drag is on the y-axis and time is on the x-axis. Therefore, the best-fit curve equation actually is $F_{Drag} = 0.2275t^2 + 0.3187t$.
- Take the definite integral of the Force of Drag with respect to time to get the area under the curve.
  - $\int_0^{9.25} F_{Drag}\, dt = \int_0^{9.25} \left(0.2275t^2 + 0.3187t\right) dt = \left[\frac{0.2275t^3}{3} + \frac{0.3187t^2}{2}\right]_0^{9.25}$

$$\Rightarrow \int_0^{9.25} F_{Drag}\, dt = \frac{0.2275\left(9.25\right)^3}{3} + \frac{0.3187\left(9.25\right)^2}{2} - \left(\frac{0.2275\left(0\right)^3}{3} + \frac{0.3187\left(0\right)^2}{2}\right) = 73.653 N \cdot s$$

- This is also equal to the Average Force of Drag times the Change In Time. So we can solve for the Average Force of Drag.

$$\int_0^{9.25} F_{Drag}\, dt = F_{Drag\,Average} \Delta t = 73.653 \Rightarrow F_{Drag\,Average} = \frac{73.653}{\Delta t} = \frac{73.653}{9.25} = 7.962 \approx \boxed{8.0N}$$

Now, I am sure some of you are wondering why we can't just use the average velocity:

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{42}{9.25} = 4.541 \frac{m}{s}$$

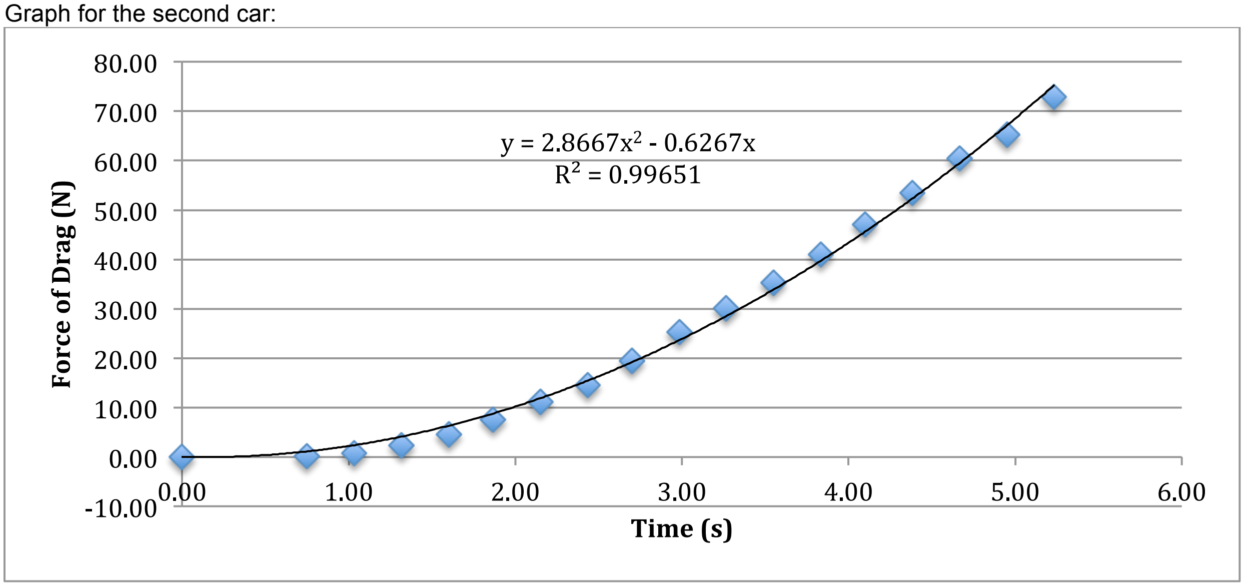
And then solve for the Average Force of Drag.

$$F_{Drag} = \frac{1}{2}\rho v^2 DA = \frac{1}{2}\left(1.2\right)\left(4.541\right)^2\left(0.25\right)\left(2.00651\right) = 6.206 \approx 6.2N \text{ (incorrect)}$$

This *incorrect* solution does not account for the fact that the velocity is squared in the force of drag equation and therefore the instantaneous force of drag does not increase linearly. Using the average velocity to solve for the average force of drag would only be correct if the force of drag increased linearly. The faster the car moves, the more incorrect this incorrect solution becomes. See below:

Note: The more chunks of time we can break the event into, the more accurate the calculation for the curve. In this particular case we were limited by the refresh rate of the speedometer.

Graph for the second car:



$$\int_0^{5.43} F_{Drag}\, dt = \int_0^{5.43} \left(2.8667t^2 - 0.6267\right) dt = \left[\frac{2.8667t^3}{3} - \frac{0.6267t^2}{2}\right]_0^{5.43}$$

$$\Rightarrow \int_0^{5.43} F_{Drag}\, dt = \frac{2.8667\left(5.43\right)^3}{3} - \frac{0.6267\left(5.43\right)^2}{2} - \left(\frac{2.8667\left(0\right)^3}{3} - \frac{0.6267\left(0\right)^2}{2}\right) = 143.750 N \cdot s$$

$$\int_0^{6.98} F_{Drag}\, dt = F_{Drag\ Average} \Delta t = 143.75 \Rightarrow F_{Drag\ Average} = \frac{143.75}{\Delta t} = \frac{143.75}{5.43} = 26.473 \approx 26N$$

Incorrect Solution:

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{42}{5.43} = 7.735 \frac{m}{s}$$

$$F_{Drag} = \frac{1}{2} \rho v^2 DA = \frac{1}{2}\left(1.2\right)\left(7.735\right)^2\left(0.25\right)\left(2.00651\right) = 18.007 \approx 18N$$

See, it’s more incorrect the faster the vehicle moves.