Flipping Physics Lecture Notes:

Graphing Instantaneous Power

Example: An 8.53 kg pumpkin is dropped from a height of 8.91 m. Will the graph of instantaneous power delivered by the force of gravity as a function of _____ be linear? If not, what would you change to make the graph linear? (a) Time, (b) Position.

The equation for instantaneous power delivered by the force of gravity is:

$$P_{F_g} = F_g v_{inst} \cos\theta = (mg) v_{inst} \cos(0) = mgv_{inst}$$

We can substitute mass times the acceleration due to gravity because that is the equation for the force of gravity. And we can substitute zero degrees for theta because the force of gravity is down and the instantaneous velocity is down and the angle between down and down is zero degrees.

What we need is an expression for the instantaneous velocity. The pumpkin is in free fall so it's acceleration equals –g or -9.81 m/s^2. Because the acceleration is constant, we can use the uniformly accelerated motion (UAM) equations. Which means the instantaneous velocity we are referring to here is the final velocity in the y-direction in the UAM equations.

Let's start with part (a): Instantaneous Power as a function of time. Therefore, we need the velocity final in the y-direction of the pumpkin in terms of time:

$$v_{fy} = v_{iy} + a_y \Delta t = 0 + (-g)(t_f - t_i) = (-g)(t_f - 0) = -gt_f$$

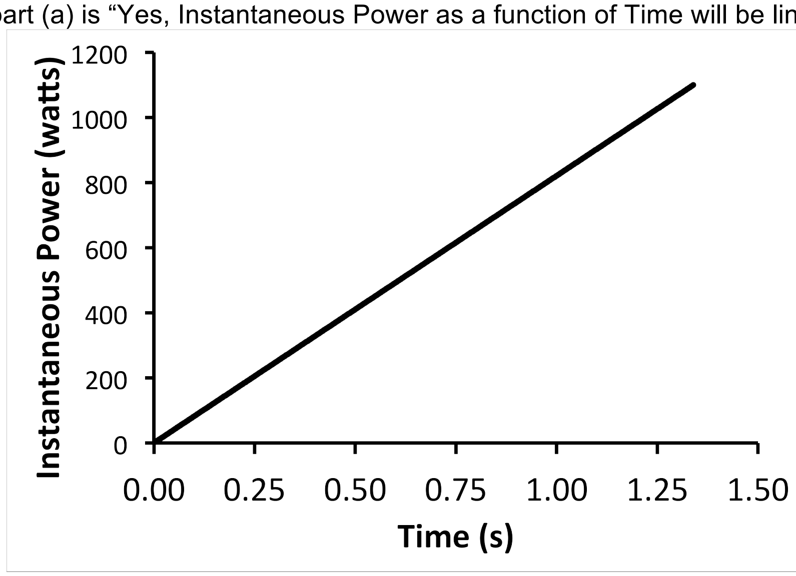
Before we plug $-gt_f$ into the equation for instantaneous power, recall that we use only the magnitude of the force and the velocity in the power equation, therefore, we use $+gt_f$ instead.

$$P_{F_g} = mg(gt_f) = mg^2 t_f \ \ \& \ \ y = (slope)x + b$$

(I use this instead of $y = mx + b$ to avoid redundant "m" variables.)

Note: With Power on the y-axis and time final on the x-axis, we should get a linear relationship with mg^2 as the slope of the line and a y-intercept of zero.

So the answer to part (a) is "Yes, Instantaneous Power as a function of Time will be linear."



$$slope = mg^2 = (8.53)(9.81^2) = 820.894 \approx 821 \frac{kg \cdot m^2}{s^4}$$

Part (b) Instantaneous Power as a function of height. Therefore, we need the velocity final in the y-direction of the pumpkin in terms of position:

$$v_{fy}^{\ 2} = v_{iy}^{\ 2} + 2a_y \Delta y = 0^2 + 2\left(-g\right)\left(y_f - y_i\right) = -2g\left(y_f - 0\right) \Rightarrow v_{fy} = \sqrt{-2gy_f}$$

Note: We have set the initial position to be zero.

Which we can plug back into the equation for instantaneous power:

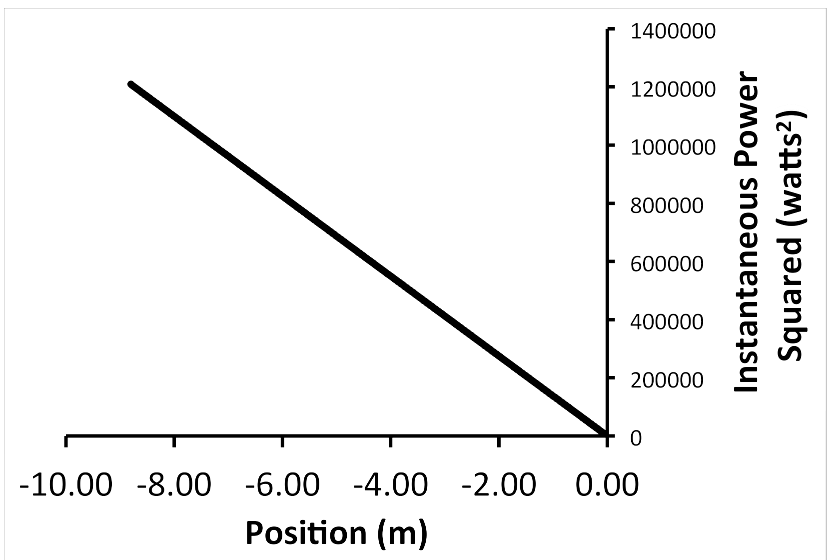
$$P_{F_g} = mgv_{inst} = mg\sqrt{-2gy_f} \ \& \ y = \left(slope\right)x + b$$

(arggg!! Two different y's. Note: "y_f" is the final position of the pumpkin and "y" represents the y-axis variable.)

Notice that, because of the square root in the equation, this will *not* yield a linear relationship between P_{F_g} and y_f. The answer to part (b) is "No, Instantaneous Power as a function of Height will not be linear." And we need to square the equation to determine what to put on the x and y axis to get a linear relationship.

$$\left(P_{F_g}\right)^2 = m^2g^2\left(-2gy_f\right) = -2m^2g^3y_f \ \& \ y = \left(slope\right)x + b$$

Therefore power squared is on the y-axis and position is on the x-axis:



$$slope = -2m^2g^3 = -\left(2\right)\left(8.53^2\right)\left(9.81^3\right) = \text{-}137384 \approx \text{-}137000\frac{kg^2 \cdot m^3}{s^6}$$