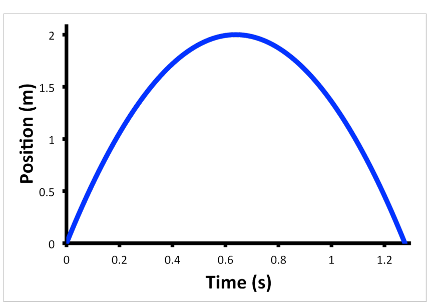
Flipping Physics Lecture Notes:

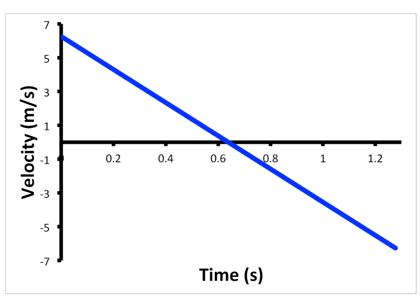
AP Physics C: Integrals in Kinematics Review (Mechanics)
https://www.flippingphysics.com/apc-integrals-kinematics-review.html

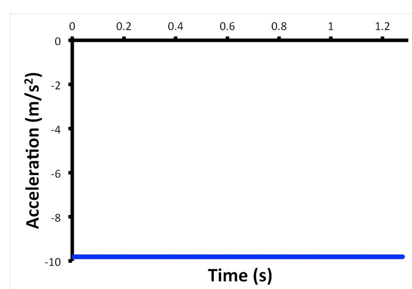
To be reviewed *after* students learn about integrals!!

- FYI: I do not teach integrals until we get to Work. By then the students who are taking calculus concurrently with AP Physics C Mechanics have had enough experience with derivatives that they only freak out a little bit when I teach them integrals. ☺

- Remember, every derivative is also an antiderivative (or an integral). For example:
  - $a = \frac{dv}{dt} \Rightarrow dv = a\,dt \Rightarrow \int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a\,dt \Rightarrow v\Big]_{v_i}^{v_f} = v_f - v_i = \Delta v = \int_{t_i}^{t_f} a\,dt$
  - The area "under" an acceleration as a function of time graph is the change in velocity of the object.
    - Remember the area "under" the curve specifically means the area between the curve and the horizontal axis where area above the horizontal axis is positive and area below the horizontal axis is negative.
- Another Example:
  - $v = \frac{dx}{dt} \Rightarrow dx = v\,dt \Rightarrow \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v\,dt \Rightarrow x\Big]_{x_i}^{x_f} = x_f - x_i = \Delta x = \int_{t_i}^{t_f} v\,dt$
  - The area "under" an velocity as a function of time graph is the change in position of the object or the displacement of the object.
- Graphs of throwing a ball upward with a positive velocity initial.
  - $v = \frac{dx}{dt}$ → Velocity is the slope of a position vs. time graph.
  - $a = \frac{dv}{dt}$ → Acceleration is the slope of a velocity vs. time graph.
  - $\Delta v = \int_{t_i}^{t_f} a\,dt$ → Change in velocity is the area "under" an acceleration as a function of time graph.
  - $\Delta x = \int_{t_i}^{t_f} v\,dt$ → Change in position or displacement, is the area "under" a velocity as a function of time graph.



- Assuming the acceleration is constant, we can derive two of the Uniformly Accelerated Motion equations. For example:
    - $a=\frac{dv}{dt}\Rightarrow dv=a\,dt\Rightarrow\int dv=\int a\,dt\Rightarrow v(t)=at+C$
    - $\Rightarrow v(0)=a(0)+C\Rightarrow v(0)=C=v_i\Rightarrow v(t)=at+v_i\Rightarrow v_f=v_i+at$
- Another example:
    - $v=\frac{dx}{dt}\Rightarrow dx=v\,dt\Rightarrow\int dx=\int v\,dt\Rightarrow x(t)=\int(v_i+at)dt=v_it+\frac{1}{2}at^2+C$
    - $\Rightarrow x(0)=v_i(0)+\frac{1}{2}a(0)^2+C\Rightarrow x(0)=C=x_i\Rightarrow x(t)=x_i+v_it+\frac{1}{2}at^2$