



Flipping Physics Lecture Notes:

AP Physics C: Momentum, Impulse, Collisions & Center of Mass Review (Mechanics)

<https://www.flippingphysics.com/apc-momentum-impulse-review.html>

- The symbol for momentum is a lowercase p. $\vec{p} = m\vec{v}$
 - Momentum is a vector!
 - $\vec{p} = m\vec{v} \Rightarrow \frac{\text{kg} \cdot \text{m}}{\text{s}}$ (not to be confused with Newtons which are $\frac{\text{kg} \cdot \text{m}}{\text{s}^2}$)
- Newton's 2nd law in terms of momentum is: $\sum \vec{F} = \frac{d\vec{p}}{dt}$
- $\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$ (the product rule)
 - $\Rightarrow \sum \vec{F} = \frac{dm}{dt}\vec{v} + m\vec{a}$ (Usually we assume the mass of the object does not change.)
- $\sum \vec{F}_{\text{external}} = \frac{d\vec{p}}{dt} = 0 \Rightarrow \sum \vec{p}_i = \sum \vec{p}_f$: When all the forces are internal to the system, the net force equals zero, the derivative of momentum as a function of time is zero, therefore the momentum does not change, therefore momentum is conserved.
 - Momentum is conserved during collisions and explosions.
- Impulse derivation: $\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \sum \vec{F} dt = d\vec{p} \Rightarrow \int_{t_i}^{t_f} \sum \vec{F} dt = \int_{p_i}^{p_f} d\vec{p} = \vec{p}_f - \vec{p}_i \Rightarrow \Delta\vec{p} = \int_{t_i}^{t_f} \sum \vec{F} dt = \vec{J}$
 - Symbol for Impulse is J and it is a vector.
 - Units for Impulse: $\vec{J} = \int_{t_i}^{t_f} \sum \vec{F} dt \Rightarrow N \cdot s$ (yes, this is the same as momentum: $\frac{\text{kg} \cdot \text{m}}{\text{s}}$)
 - $N \cdot s = \left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right) s = \frac{\text{kg} \cdot \text{m}}{\text{s}}$
 - Impulse is the area "under" a force as a function of time curve.
 - Not to be confused with the equation for work: $W = \int_{x_i}^{x_f} F_x dx$
 - Impulse approximation says $\sum \vec{F} \approx \vec{F}_{\text{impact}}$
 - Therefore, the impulse approximation says: $\vec{F}_{\text{impact}} = \frac{d\vec{p}}{dt}$
 - Impulse, J, and Impact Force often get confused. Please note they are different!
 - Can also use the average force and change in time to determine Impulse: $\vec{J} = \vec{F}_{\text{average}} \Delta t$
 - This creates a rectangle with the same area as $\vec{J} = \int_{t_i}^{t_f} \sum \vec{F} dt$

Type of Collision	Is Momentum Conserved?	Is Kinetic Energy Conserved?
Elastic	Yes	Yes
Inelastic	Yes	No

- Collisions between hard spheres are “nearly” elastic and therefore are generally considered to be elastic in physics classes.
- “Perfectly Inelastic” Collisions are where the objects stick to one another. Sometimes they are called “Completely Inelastic” or “Totally Inelastic”. These terms all mean the same thing.
- Most collisions are actually Inelastic.
- Center of mass of a system of particles:
$$\mathbf{x}_{cm} = \frac{\sum m_i \mathbf{x}_i}{\sum m_i} = \frac{m_1 \mathbf{x}_1 + m_2 \mathbf{x}_2 + \dots}{m_1 + m_2 + \dots}$$
 - x is the distance from a zero reference line; usually the origin.
 - Velocity of a system of particles:
$$\mathbf{v}_{cm} = \frac{d\mathbf{x}_{cm}}{dt} = \frac{d}{dt} \left(\frac{\sum m_i \mathbf{x}_i}{\sum m_i} \right) = \frac{\sum m_i d\mathbf{x}_i}{\sum m_i dt} = \frac{\sum m_i \mathbf{v}_i}{\sum m_i}$$
 - Do the same thing with acceleration:
$$\mathbf{a}_{cm} = \frac{d\mathbf{v}_{cm}}{dt} = \frac{d}{dt} \left(\frac{\sum m_i \mathbf{v}_i}{\sum m_i} \right) = \frac{\sum m_i \mathbf{a}_i}{\sum m_i}$$
- Center of mass of an object with shape:
$$\mathbf{r}_{cm} = \frac{1}{m_{total}} \int \mathbf{r} dm$$
 (not on AP equation sheet)
 - The position of the center of mass of an object with shape equals one over the total mass of the object times the integral with respect to mass of the position of all of the infinitesimally small pieces of the object, which are called dm , relative to a zero-reference line.
 - If you prefer:
$$\mathbf{x}_{cm} = \frac{1}{m_{total}} \int \mathbf{x} dm$$
 - Volumetric Mass Density: $\rho = \frac{m}{V}$ (not on AP equation sheet)
 - Surface Mass Density: $\sigma = \frac{m}{A}$ (not on AP equation sheet)
 - Linear Mass Density: $\lambda = \frac{m}{L}$ (not on AP equation sheet)