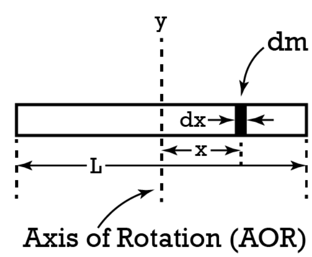
Flipping Physics Lecture Notes:

AP Physics C: Rotational Dynamics Review – 1 of 2 (Mechanics)

- A rigid object with shape is rotating. Every piece of this object has kinetic energy. The total kinetic energy is the sum of all of the kinetic energies of every small piece of the object:

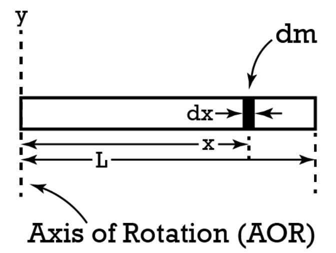
$$KE_t = \sum_i KE_i = \sum_i \frac{1}{2} m_i \left(v_i\right)^2 = \sum_i \frac{1}{2} m_i \left(r_i \omega_i\right)^2 = \sum_i \frac{1}{2} m_i r_i^2 \omega_i^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$

  - This uses $v_t = r\omega$ and that every part of the object has the same angular velocity, ω
  - $KE_{rotational} = \frac{1}{2} I \omega^2$: Rotational Kinetic Energy of a rigid object with shape or a system of particles that is not changing shape.
- $I = \sum_i m_i r_i^2$ where I is called the Moment of Inertia or "Rotational Mass".
  - This is the Moment of Inertia for a system of particles.
  - Units for Moment of Inertia: $I = \sum_i m_i r_i^2 \Rightarrow kg \cdot m^2$
- Moment of Inertia for a rigid object with shape: $I = \lim_{\Delta m \to 0} \sum r_i^2 \Delta m_i \Rightarrow I = \int r^2\, dm$
  - Not to be confused with the equation for the center of mass of a rigid object with shape: $r_{cm} = \frac{1}{m_{total}} \int r\, dm$
- Deriving the Moment of Inertia of a Uniform Thin Hoop about its Cylindrical Axis
  - $I_z = \int r^2\, dm = R^2 \int dm = R^2 m \Rightarrow I_{cm} = mR^2$
  - "Thin" means all of the dm's are located a distance R from the center of mass.
  - "Uniform" means the hoop is of a constant density.
  - "Cylindrical Axis" means the line through the center of the hoop and normal to the plane of the hoop.
- Deriving the Moment of Inertia of a Uniform Rigid Rod about its Center of Mass
  - $\lambda = \frac{m}{L} = \frac{dm}{dx} \Rightarrow dm = \lambda\, dx \Rightarrow dm = \frac{m}{L}\, dx$
    - m is the total mass of the rod
    - L is the total length of the rod
    - λ is the linear mass density of the rod, which is constant in this "uniform" rod.



  - $I_y = \int r^2\, dm = \int r^2 \frac{m}{L} dx = \frac{m}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\, dx = \frac{m}{L}\left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}}$
  - $\Rightarrow I_y = \frac{m}{L}\left[\frac{\left(\frac{L}{2}\right)^3}{3} - \frac{\left(-\frac{L}{2}\right)^3}{3}\right] = \frac{m}{L}\left[\frac{L^3}{24} + \frac{L^3}{24}\right] = \frac{m}{L}\left[\frac{2L^3}{24}\right] = \boxed{\frac{1}{12} mL^2}$

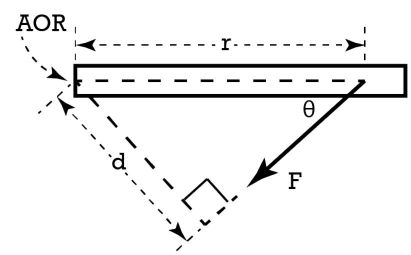
- Deriving the Moment of Inertia of a Uniform Rigid Rod about one end
  - This is the same as before, only with different limits …
  - $I_y = \frac{m}{L}\int_0^L x^2\,dx \Rightarrow \frac{m}{L}\left[\frac{x^3}{3}\right]_0^L = \frac{m}{L}\frac{L^3}{3} = \boxed{\frac{1}{3}mL^2}$



- The Parallel-Axis Theorem: $I = I_{cm} + mD^2$
  - Only true for objects with constant density.
  - m is the total mass of the rigid, constant density object.
  - D is the distance from the center of mass of the object to the new axis of rotation.
  - Not on the AP equation sheet.
- Example: Moment of Inertia of a Uniform Rigid Rod about its end.
  - Known for Uniform Rigid Rod: $I_{cm} = \frac{1}{12}mL^2$

$$I_{end} = I_{cm} + mD^2 = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + \frac{1}{4}mL^2 = \left(\frac{1}{12}+\frac{3}{12}\right)mL^2 = \frac{4}{12}mL^2 = \boxed{\frac{1}{3}mL^2}$$

- Example: Moment of Inertia of a Uniform Thin Hoop about its Rim.
  - Known for Uniform Thin Hoop about its Center of Mass: $I_{cm} = mR^2$
  - $I_{rim} = I_{cm} + mD^2 = mR^2 + mR^2 = \boxed{2mR^2}$
- Torque: $\tau = rF\sin\theta$
  - This is the magnitude of the torque. Torque is a vector.
  - r is the distance from the axis of rotation to the location on the object the force is applied.
  - F is the magnitude of the force.
  - θ is the angle between r and F.
  - $\sin\theta = \frac{O}{H} = \frac{d}{r} \Rightarrow d = r\sin\theta$ is the "moment arm" or "lever arm" or "effective distance"



  - Units for torque are $N \cdot m$
    - Not to be confused with the units for energy, joules, even though joules are also $N \cdot m$.
- But "What is torque?" Torque is the rotational equivalent of force. Force is the ability to cause an acceleration of an object. Torque is the ability of a force to cause an *angular* acceleration of an object.
- The rotational form of Newton's Second Law: $\sum \vec{F} = m\vec{a} \Rightarrow \sum \vec{\tau} = I\vec{\alpha}$
  - Must identify axis of rotation when summing the torques.
  - Must identify what objects you are summing the torque on.
    - Note: The angular acceleration of each object around the axis of rotation must be the same.
  - Must identify the direction of positive rotation.
  - Now that we have defined Moment of Inertia, pulleys can have mass. When pulleys have mass the force of tension on either side of a pulley are *not* the same!
- Right Hand Rule for direction of torque
  - Don't be too cool for the right hand rule. Limber Up!
  - Use your right hand.
  - Fingers start at the axis of rotation.
  - Point fingers along direction of "r".
  - Curl fingers along the direction of "F".
  - Thumb points in the direction of the torque.

- Rolling Without Slipping: $v_{cm} = R\omega$ & $a_{cm} = R\alpha$
    - Just like $v_t = r\omega$ & $a_t = r\alpha$ only …
        - R is the radius of the solid object
        - These are for the center of mass of the object, not the tangential quantities.
    - Neither of these are on the AP equation sheet.
    - FYI: Rolling *With* Slipping: $v_{cm} \neq R\omega$ & $a_{cm} \neq R\alpha$
    - When an object is rolling without slipping it has **both** translational and rotational kinetic energies!!