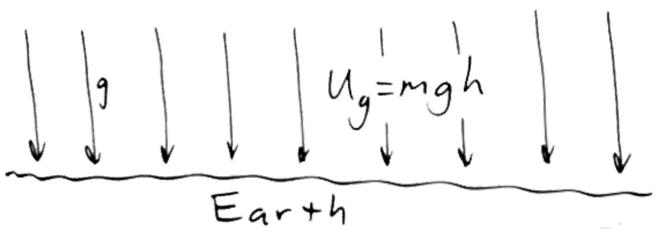
Flipping Physics Lecture Notes:

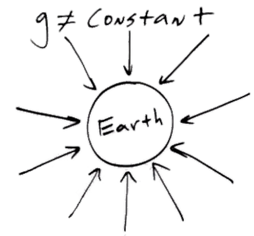
AP Physics C: Universal Gravitation Review (Mechanics)
https://www.flippingphysics.com/apc-universal-gravitation-review.html

- The Force of Gravity or Weight of an object: $F_g = mg$
  - A subscript is missing: $F_g = m_o g$ where "o" stands for "object".
  - All forces require two objects. This equation is the force of gravitational attraction which exists between the object and a planet.
    - For us, the planet usually is the Earth. $g_{Earth} = 9.81\frac{m}{s^2}$
- Any two objects which have mass have a force of gravitational attraction between them. This force is determined using Newton's Universal Law of Gravitation. (The Big G Equation)
  - $F_g = \frac{Gm_1m_2}{r^2}$: G is the Universal Gravitational Constant. $G = 6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}$
    - Requires ***two*** objects: mass 1 and mass 2.
    - r is ***not*** the radius by definition. r is the distance between the centers of mass of the two objects. Yes, sometimes r is the radius.
    - This equation is the magnitude of the force of gravitational attraction. The force is always directed toward the other object.
  - Thank you to one of my Quality Control team members, Frank Geshwind, for pointing out that I should have talked about the vector form of this equation: $\vec{F}_g = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$
    - Note: $\hat{r}_{12}$ is the unit vector from object 1 to object 2.
    - Some textbooks even use this equation: $\vec{F}_g = -\frac{Gm_1m_2}{r^3}\vec{r}_{12}$
      - Note the subtle change here. $\vec{r}_{12}$ is no longer the unit vector and therefore the cube of r needs to be in the denominator. ☺
- Setting the two equations for the Force of Gravity acting on an object on the surface of the Earth equal to one another yields this:
  - $F_g = F_g \Rightarrow m_o g = \frac{Gm_o m_E}{r^2} \Rightarrow g_{Earth} = \frac{Gm_E}{\left(R_{Earth} + Altitude\right)^2}$
- As long as $R_{Earth} \gg \Delta Altitude$ the $g_{Earth} \approx constant$.
  - In other words, close to the surface of the planet, the acceleration due to gravity can be considered to be constant. We live in a constant gravitational field.

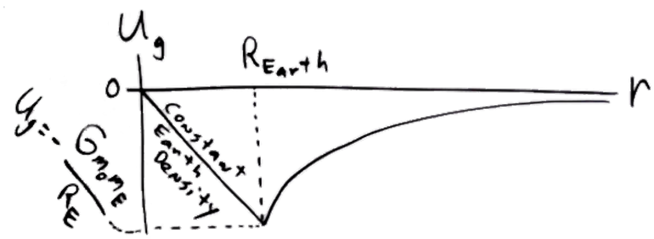


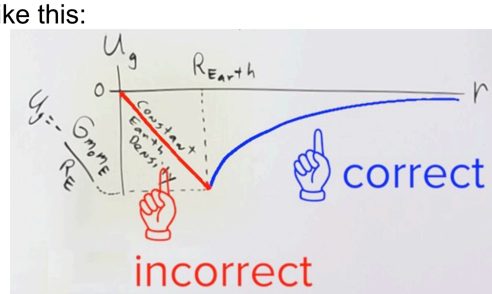
  - The gravitational potential energy is then constant: $U_g = mgh$

- When viewed from a frame of reference which is not on the surface of the planet, the acceleration due to gravity is *not* constant and we need a different equation:
  - Universal Gravitational Potential Energy: $U_g = -\frac{Gm_1m_2}{r}$
    - This equation assumes a zero line which is infinitely far away. ($r \approx \infty$)
    - This means Universal Gravitational Potential Energy *can never be positive.*
    - This equation requires two objects.
    - Note: This equation is not $F_g = \frac{Gm_1m_2}{r^2}$ r is *not* squared.



- The gravitational potential energy which exists between an object and the Earth in terms of the objects distance, r, from the center of the Earth looks like this:





  - Assumes constant density Earth. (Which is not true, however, we often assume it is.)
  - Be aware the graph as shown above left has an incorrect part:
    - This cannot be correct because this implies an object would experience a change in gravitational potential energy of zero.
      - $\Delta U_g = U_f - U_i = 0 - 0 = 0$ (This makes no sense.)
    - This implies it takes zero energy to move an object from the center of a planet to infinitely far from the planet.
      - $W_{F_a} = \Delta ME = \Delta U_g = 0$ (Again, this makes no sense.)
    - This would mean the planet has a binding energy of zero; still makes no sense.
    - I will be releasing a video with the correct solution soon.
  - The Gravitational Potential Energy which exists between the two objects when the object is on the surface of the Earth is: $U_g = -\frac{Gm_om_E}{R_E}$
- What is the minimum amount of work necessary to completely remove an object from a planet if the object is resting on the surface of the planet? This is called the Binding Energy. Assume the object is moved infinitely far away, has zero velocity when it gets there, and there is no friction.

$$\Delta E_{system} = \sum T \Rightarrow \Delta ME + \Delta E_{\text{internal}} = W_{F_a}$$

$$\Rightarrow W_{F_a} = ME_f - ME_i + 0 = 0 - U_{gi} = 0 - \left(-\frac{Gm_1m_2}{r}\right) = \frac{Gm_om_p}{R_p} \text{ (Binding Energy)}$$

- What is the minimum velocity to launch an object off the Earth and have it never return? This is called Escape Velocity. Assume no atmosphere and no Earth rotation. Note: Mechanical Energy is conserved:

$$ME_i = ME_f \Rightarrow U_{gi} + KE_i = 0 \Rightarrow -\frac{Gm_om_E}{R_E} + \frac{1}{2}m_ov_i^2 = 0 \Rightarrow \frac{1}{2}v_i^2 = \frac{Gm_E}{R_E} \Rightarrow v_{escape} = \sqrt{\frac{2Gm_E}{R_E}}$$

- What is the total mechanical energy of an object in circular orbit?

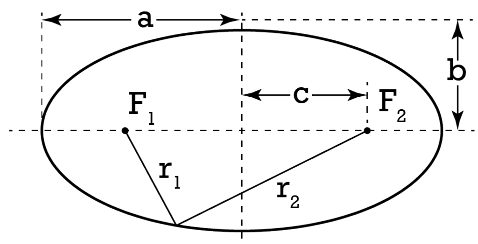
$$ME_{total} = U_g + KE = -\frac{Gm_o m_{planet}}{r} + \frac{1}{2}m_o v_o^2$$

The only force acting on the object moving in circular orbit is the force of gravity which acts inward. So we can sum the forces on the object in the in direction:

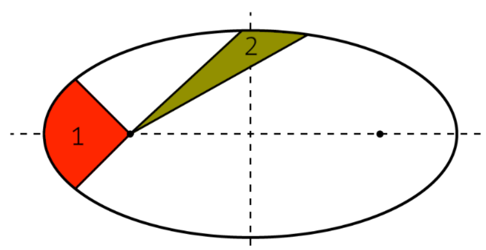
$$\sum F_{in} = F_g = ma_c \Rightarrow \frac{Gm_o m_p}{r^2} = m_o \frac{v_o^2}{r} \Rightarrow \frac{Gm_o m_p}{2r} = \frac{1}{2}m_o v_o^2 = KE_o$$

$$ME_{total} = -\frac{Gm_o m_p}{r} + \frac{Gm_o m_p}{2r} = \frac{Gm_o m_p}{r}\left(-1+\frac{1}{2}\right) = \frac{Gm_o m_p}{r}\left(-\frac{1}{2}\right) = -\frac{Gm_o m_p}{2r}$$

- Kepler's 3 laws. I find having a basic understanding of the his first two laws is adequate, however, you need to know how to derive Kepler's 3rd law.
- Kepler's 1st Law is that orbits are elliptical and defined as such:



  - Two foci at F_1 and F_2.
  - Each focus is located a distance c from the center of the ellipse.
  - Semimajor axis of length a. (Major axis of length 2a)
  - Semiminor axis of length b. (Minor axis of length 2b)
  - $r_1 + r_2 = 2a$
  - $a^2 = b^2 + c^2$
  - Eccentricity of an ellipse is defined as c/a. For a circle c = 0 therefore eccentricity = 0.
  - The eccentricity of the Earth's orbit is 0.017, which means its orbit is nearly circular.
- Kepler's 2nd Law states that a line between the sun and the planet sweeps out an equal area over an equal time interval.



  - The result of this is that the closer an object is to the planet during an orbit, the faster its orbital speed will be.
- Let's derive Kepler's 3rd law: We assume circular orbit. The only force acting on the orbital object is the force of gravity acting inward. Sum the forces in the in direction:

$$\sum F_{in} = F_g = ma_c \Rightarrow \frac{Gm_o m_p}{r^2} = m_o r\omega^2 \Rightarrow \frac{Gm_p}{r^3} = \omega^2 = \left(\frac{\Delta\theta}{\Delta t}\right)^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2}{T^2} \Rightarrow T^2 = \left(\frac{4\pi^2}{Gm_p}\right)r^3$$