



Flipping Physics Lecture Notes:

Deriving Escape Velocity

Example: Determine the escape velocity of planet Earth. Assume no air resistance and no planet rotation.

Knowns: $m_{Earth} = 5.97 \times 10^{24} \text{ kg}$; $R_{Earth \text{ average}} = 6.37 \times 10^6 \text{ m}$

The escape velocity of a planet is the minimum speed at which an object can be launched such that it would take an infinite amount of time to slow the object to a stop.

No force applied and no air resistance means we can use conservation of mechanical energy: $ME_i = ME_f$

Set the initial point at the surface of the planet and the final point infinitely far away from the planet.

The object starts with gravitational potential energy and kinetic energy. It ends with zero gravitational potential energy, because it is infinitely far away from the planet. It also ends with zero kinetic energy because this is the *minimum speed*. If the speed were anything greater, the object would end with some kinetic energy.

$$\Rightarrow U_{gi} + KE_i = 0$$

Now we can substitute in equations.

$$\Rightarrow -\frac{Gm_1m_2}{r} + \frac{1}{2}mv_i^2 = 0$$

Add gravitational potential energy to both sides and substitute in variables for mass, radius, and velocity.

$$\Rightarrow \frac{1}{2}m_o v_{oi}^2 = \frac{Gm_o m_E}{R_E}$$

Everyone brought the mass of the object to the party and solve for the velocity of the object initial, which is the escape velocity.

$$\Rightarrow \frac{1}{2}v_{oi}^2 = \frac{Gm_E}{R_E} \Rightarrow v_{oi} = v_{\text{escape}} = \sqrt{\frac{2Gm_E}{R_E}}$$

Please notice the mass of the object is irrelevant.

Substitute in numbers.

$$\Rightarrow v_{\text{escape}} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.96 \times 10^{24})}{6.37 \times 10^6}} = 11,181.38 \frac{m}{s} \times \frac{1km}{1000m} \approx \boxed{11.2 \frac{km}{s}}$$

$$\Rightarrow v_{\text{escape}} = 11,181.38 \frac{m}{s} \times \frac{3600s}{1hr} \times \frac{1mi}{1609m} = 25,017.38 \approx 25,000 \frac{mi}{hr} \approx \boxed{2.50 \times 10^4 \frac{mi}{hr}}$$

♥ <https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html>