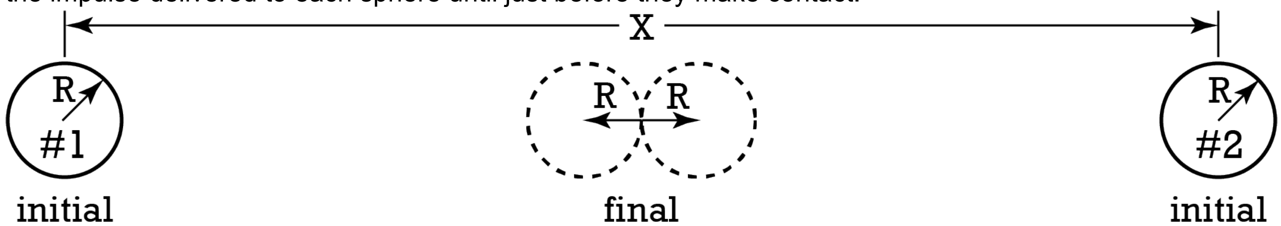
Flipping Physics Lecture Notes:

Impulse for Two Objects being Attracted to One Another

Example: In a universe devoid of anything else, two identical spheres of mass, m, and radius, R, are released from rest when they have a distance between their centers of mass of X. Find the magnitude of the impulse delivered to each sphere until just before they make contact.



Knowns: $v_{1i} = v_{2i} = 0;\ m_1 = m_2 = m;\ r_1 = r_2 = R;\ \text{initial center of mass distance} = X;\ J = ?$

The equation for Impulse is: $\vec{J} = \Delta\vec{p} = \text{Area under Force vs. Time curve}$

Area under Force vs. Time curve is unhelpful in this situation. (There is no Force vs. Time curve.)

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = m\vec{v}_f - m(0) = m\vec{v}_f$$

All we need is the final velocity of one sphere. In fact, we only need the magnitude of the final velocity.

No work done by a Force Applied and no energy converted to heat or sound via friction → Conservation of Mechanical Energy: $ME_i = ME_f$; The initial and final points are already identified in the drawing. The only type of initial mechanical energy is the universal gravitational potential energy. Finally both objects have kinetic energy and universal gravitational potential energy.

$$\Rightarrow U_{gi} = KE_{1f} + KE_{2f} + U_{gf}$$

Substitute in equations and realize one expression for the universal gravitational potential energy includes the universal gravitational potential energy for both spheres.

$$\Rightarrow -\frac{Gm_1m_2}{r_i} = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

Substitute in values: $r_i = X;\ m_1 = m_2 = m;\ v_{1f} = v_{2f} = v_f;\ r_f = 2R$ (because the final distance between the centers of mass includes two radii.) Everybody brought mass to the party!

$$\Rightarrow -\frac{Gmm}{X} = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_f^2 - \frac{Gmm}{2R} \Rightarrow -\frac{Gm}{X} = \frac{1}{2}v_f^2 + \frac{1}{2}v_f^2 - \frac{Gm}{2R}$$

And now some algebra. $\Rightarrow \frac{Gm}{2R} - \frac{Gm}{X} = v_f^2 = Gm\left(\frac{1}{2R} - \frac{1}{X}\right) \Rightarrow v_f = \sqrt{Gm\left(\frac{1}{2R} - \frac{1}{X}\right)}$

And back to the equation for the magnitude of the impulse.

$$J = mv_f = m\sqrt{Gm\left(\frac{1}{2R} - \frac{1}{X}\right)} = \boxed{\sqrt{Gm^3\left(\frac{1}{2R} - \frac{1}{X}\right)}}$$