Flipping Physics Lecture Notes:

Simple Harmonic Motion - Velocity and Acceleration Equation Derivations

Previously[♥] we derived the equation on the AP Physics 1 equation sheet for an object moving in simple harmonic motion: $x(t) = A\cos(2\pi ft)$.

In order to derive the equations for velocity and acceleration, let's get position in terms of angular frequency: $f = \frac{1}{T}$ & $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$ therefore $x(t) = A\cos(2\pi ft) = A\cos(\omega t)$.

Let's add the phase constant that shifts the wave along the horizontal axis: $x(t) = A\cos(\omega t + \phi)$.

Velocity is the derivative of position as a function of time. I know some of you might not have taken calculus yet and might not understand derivatives. Realize you need derivatives to derive velocity and acceleration simple harmonic motion equations. Some of this math might go over your heads, however, it is still useful to get some exposure to. ☺

Uses chain rule.

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[A\cos(\omega t + \phi)\right] = A\frac{d}{dt}\left[\cos(\omega t + \phi)\right] = A\left[-\sin(\omega t + \phi)\right]\left[\frac{d}{dt}(\omega t + \phi)\right]$$

$$\Rightarrow v(t) = -A\sin(\omega t + \phi)\omega \Rightarrow v(t) = -A\omega\sin(\omega t + \phi)$$

Note: Because $-1 \le \sin\theta \le 1 \rightarrow v_{max} = A\omega$

The derivation of acceleration is very similar:

Again, uses chain rule.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[-A\omega\sin(\omega t + \phi)\right] = -A\omega\frac{d}{dt}\left[\sin(\omega t + \phi)\right] = -A\omega\left[\cos(\omega t + \phi)\right]\left[\frac{d}{dt}(\omega t + \phi)\right]$$

$$\Rightarrow a = -A\omega\left[\cos(\omega t + \phi)\right](\omega) \Rightarrow a(t) = -A\omega^2\cos(\omega t + \phi)$$

Again note: Because $-1 \le \cos\theta \le 1 \rightarrow a_{max} = A\omega^2$

Remember: Because the derivation of these equations requires theta to be in radians, all angles in these equations need to be in radians and your calculator needs to be in radian mode when using equations for position, velocity, and acceleration as a function of time in simple harmonic motion.

[♥] https://www.flippingphysics.com/shm-position.html