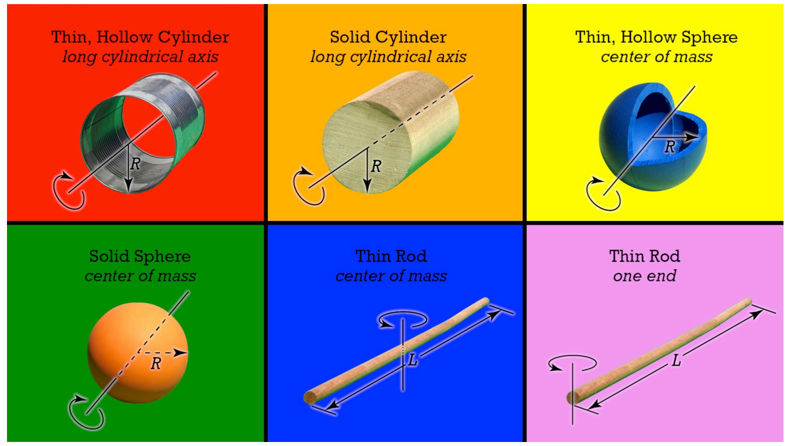
Flipping Physics Lecture Notes:

Moments of Inertia of Rigid Objects with Shape



We are going to discuss six different equations for moments of inertia of rigid objects with constant density:

1. Thin, hollow cylinder about its long cylindrical axis.
2. Solid cylinder about its long cylindrical axis.
3. Thin, hollow sphere about is center of mass.
4. Solid sphere about its center of mass.
5. Thin rod about its center of mass.
6. Thin rod about one end.

A "rigid" object will not easily change shape.
Please do not miss the fact that the density of these objects has to be constant.

In a typical calculus based physics class you would derive many of these moments of inertia, however, for this algebra based class, we will only discuss their relative moments of inertia. In my opinion it is not worth memorizing these equations, however, understanding why they have their relative values is *very* worthwhile.

Let's start with the last two: A thin rod about (a) its center of mass and (b) one end. First off realize a "thin" rod means the radius of the rod is very small relative to the length of the rod, so we consider the rod to essentially be one-dimensional. Both moments of inertia for the thin rod are a fraction times ML^2. Where "M" is the mass of the rod and "L" is the length of the rod.

The equation for the moment of inertia of a system of particles is: $I = \sum_i m_i (r_i)^2$.

The calculus version of moment of inertia of a rigid object with shape and constant density is: $I = \int r^2 dm$

The moment of inertia of the thin rod about its center of mass is: $I_{rod@center} = \frac{1}{12}ML^2$

Because some of the mass is farther from the axis of rotation when the rod is rotated about its end rather than about its center of mass and we are squaring the distance the pieces of the rigid object are from the axis of rotation in the moment of inertia equation, we would expect the moment of inertia about one end to be greater in value than about its center.♥ $I_{rod@end} = \frac{1}{3}ML^2$ and $\frac{1}{3} > \frac{1}{12}$, so it works out.

The other four moments of inertia are a fraction times MR^2. Let's start with the thin, hollow cylinder about its long cylindrical axis. Again, the term "thin" here means the thickness of the hollow cylinder is very small relative to the radius of the cylinder, therefore we can consider the hollow cylinder to be essentially a two dimensional object where every piece of the thin, hollow cylinder is a distance R from the axis of rotation. That means every "r" value equals R, the radius of the hollow cylinder. Therefore the moment of inertia of a thin, hollow cylinder about its long cylindrical axis is MR^2. $I_{hollow\ cylinder} = MR^2$ (note the "fraction" here is 1.)

♥ My apologies. The footnoted, 70-word sentence may be the longest I have ever written.

Because more of the mass of the solid cylinder is closer to the axis of rotation than for the thin, hollow cylinder, you would expect the fraction for the equation for the moment of inertia of a solid cylinder to be less than for a thin, hollow cylinder. $I_{solid\ cylinder} = \frac{1}{2}MR^2$ and $\frac{1}{2} < 1$, so it works out.

Notice neither of these two moments of inertia depend on the length of the cylinder. That means the equation for the moment of inertia of a solid disk is the same as for a solid cylinder. And the equation for the moment of inertia of a thin, hollow cylinder is the same as for a thin ring.

Next let's discuss the moment of inertia of a solid sphere about its center of mass. Compared to the solid cylinder, more of the solid sphere's mass is concentrated near its axis of rotation. Therefore, we would expect the fraction for the equation for the moment of inertia of a solid sphere to be less than for a solid cylinder. $I_{solid\ sphere} = \frac{2}{5}MR^2$ and $\frac{2}{5} < \frac{1}{2}$, so it works out.

Last is the moment of inertia of a thin, hollow sphere about its center of mass. Compared to the solid cylinder, a hollow sphere has a larger proportion of its mass located farther from the axis of rotation, so we would expect the fraction for the equation for the moment of inertia of a thin, hollow sphere about its center of mass to be more than for a solid cylinder. However, compared to the thin, hollow cylinder, a hollow sphere has a smaller proportion of its mass located farther from the axis of rotation, so we would expect the fraction for the equation for the moment of inertia of a thin, hollow sphere about its center of mass to be less than for a thin, hollow cylinder. $I_{hollow\ sphere} = \frac{2}{3}MR^2$ and $\frac{1}{2} < \frac{2}{3} < 1$, so it works out.

Again, please *do not memorize* these equations. Instead, **understand** why they have their relative values.