Flipping Physics Lecture Notes:

Torque Introduction

Two general types of motion:

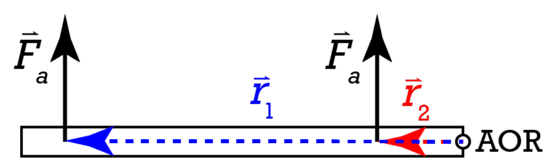
- Translational Motion: The center of mass moves from one location to another location.
  - Caused by a net force.
  - Force is the ability to cause an acceleration of an object.
- Rotational Motion: The object moves in circular motion about its center of mass.
  - Caused by a net torque
  - Torque is the ability of a force to cause an angular acceleration of an object.

The equation for torque is $\boxed{\vec{\tau} = \vec{r}\vec{F}\sin\theta}$

- The symbol for torque is the lowercase Greek letter tau, τ.
- F is the force causing the torque.
- The equation is sometimes given as $\vec{\tau} = \vec{r}_{\perp}\vec{F}$
- $\vec{r}_{\perp}$ is called the "moment arm" or "lever arm" and $\vec{r}_{\perp} = \vec{r}\sin\theta$
- $\vec{r}$ is the position vector from the axis of rotation to where the force is applied to the object.
- θ is the angle between the direction of the force and the direction of $\vec{r}$.
- Torque is a vector, which means it has both magnitude and direction.
  - We will talk about direction in detail in the next lesson.

Everything you ever needed to know about torque, you already know, because you have opened many doors. I know that all seems a bit confusing, so let's walk through some example problems involving a door. When you approach a door you find the handle and the handle is always located far from the hinge. That is because the hinge is the axis of rotation. The distance from the axis of rotation to the location the force is applied (the handle) is the magnitude of the variable "r".

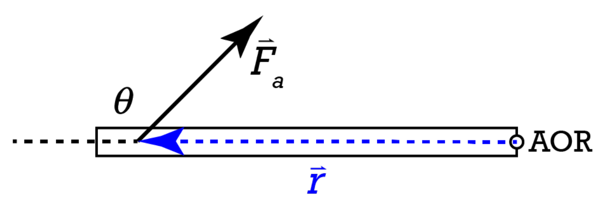
Let's start by assuming you are always pushing or pulling on the door at a 90 degree angle to the door:



$$\sin(90°) = 1 \Rightarrow \vec{r}_{\perp} = \vec{r}\sin\theta = \vec{r}\sin(90) = \vec{r} \Rightarrow \vec{\tau} = \vec{r}\vec{F}$$

In this special case "r" times the force equals the torque. The handle is far from the axis of rotation so the torque associated with the force is large. When we push on the door with the same force near the axis of rotation, the "r" value is small and therefore, even with the same force, the torque is small, which means the ability to cause an angular acceleration of the door is small.

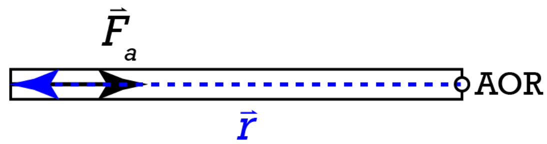
What if we push on the door at an angle which is not 90° like a 45° angle? Then, because of the shape of the sine curve, the torque associated with this force and "r" value will be reduced. In other words, because $\sin(90°) = 1$, an angle of 90° between "r" and the force, will produce the largest torque.

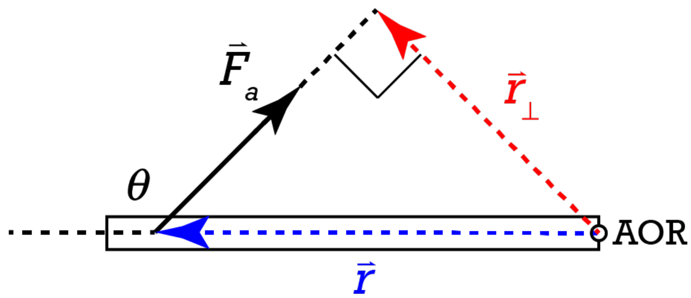


What if we push on the door at an angle of 0° or 180°?

$$\sin\left(0^\circ\right) = \sin\left(180^\circ\right) = 0 \Rightarrow \vec{\tau} = \vec{r}\vec{F}\sin\theta = \vec{r}\vec{F}\left(0\right) = 0$$

This results in zero torque or zero ability to cause an angular acceleration of the door.





But what is $\vec{r}_\perp$, the "moment arm" or "lever arm"? Going back to pushing on the door at an angle.

Illustration. Notice "r" is the hypotenuse of a right triangle that has two sides which are the force and the moment arm. In this example, the moment arm has a smaller magnitude than "r". The only way the moment arm and "r" have the same value is if the angle is 90°. In other words, a 90° angle results in the largest torque, assuming the force and "r" value are the same. Notice also that if the angle is 0°, then the moment arm equals zero.

The units for torque are $\vec{\tau} = \vec{r}\vec{F}\sin\theta \Rightarrow m \cdot N \Rightarrow N \cdot m$.

Typically they are given as netwon meters instead of joules to differentiate torque from work and energy.