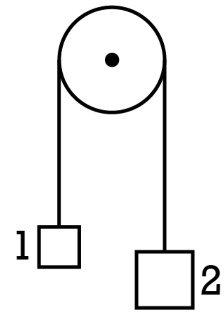
Flipping Physics Lecture Notes:

2 Masses on a Pulley - Torque Demonstration

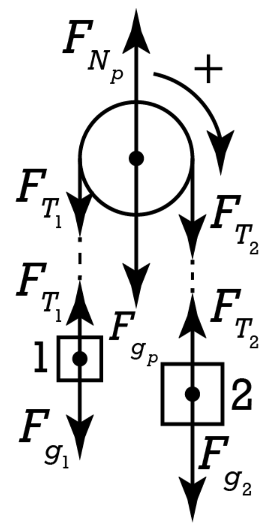
Example: 0.100 kg and 0.200 kg masses hang from either side of a frictionless pulley with a rotational inertia of 0.0137 kg·m^2 and radius of 0.0385 m. (a) What is the angular acceleration of the pulley? (b) What is the force of tension in each string?



Knowns:

$$m_1 = 0.100kg;\ m_2 = 0.200kg;\ I = 0.0137kg\cdot m^2;\ R = 0.0385m;\ \alpha = ?$$

Start with free body diagrams of the forces acting on the two masses and the pulley.



Note: If the pulley had no friction and no mass, the two forces of tension on either side of the pulley would be the same. In this example the pulley has mass and therefore rotational inertia, therefore, the forces of tension in the strings on either side of the pulley are not equal.

Let's start by summing the torques on the pulley with its axle as the axis of rotation. Because mass 2 is greater than mass 1, mass 2 should apply a larger torque on the pulley and cause the pulley to rotate in the clockwise, or into the board, direction. Therefore, let's define clockwise, or into the board, as positive. Notice that, because they both act on the axis of rotation, neither the force normal nor the force of gravity acting on the pulley will cause a torque on the pulley.

$$\sum \vec{\tau}_{\substack{pulley \\ AoR\ @\ Axle}} = \vec{\tau}_2 - \vec{\tau}_1 = I\vec{\alpha} \Rightarrow r_2 F_{T_2}\sin\theta_2 - r_1 F_{T_1}\sin\theta_1 = I\alpha$$

$$\Rightarrow RF_{T_2}\sin(90) - RF_{T_1}\sin(90) = I\alpha \Rightarrow RF_{T_2} - RF_{T_1} = I\alpha$$

(put in equation holster!)

We do not know either force of tension so we cannot currently solve for angular acceleration.

Sum the forces on mass 1:

$$\sum F_{\substack{mass\ 1 \\ +\ direction}} = F_{T_1} - F_{g_1} = m_1 a_1 \Rightarrow F_{T_1} = F_{g_1} + m_1 a_1 = m_1 g + m_1 a_1$$

(put in equation holster!)

Sum the forces on mass 2:

$$\sum F_{\substack{mass\ 2 \\ +\ direction}} = -F_{T_2} + F_{g_2} = m_2 a_2 \Rightarrow F_{T_2} = F_{g_2} - m_2 a_2 = m_2 g - m_2 a_2$$

(put in equation holster!)

Notice force of tension 2 may be *up,* however, according to the positive direction we defined, force of tension 2 is acting in the negative direction!

Combine all three equations!!!

$$I\alpha = RF_{T_2} - RF_{T_1}\ \&\ F_{T_1} = m_1 g + m_1 a_1\ \&\ F_{T_2} = m_2 g - m_2 a_2$$

$$\Rightarrow I\alpha = R(m_2 g - m_2 a_2) - R(m_1 g + m_1 a_1)$$

Notice we can relate the linear accelerations to angular acceleration: $a_1 = a_2 = a_t = r\alpha = R\alpha$

$$\Rightarrow I\alpha = R(m_2 g - m_2 R\alpha) - R(m_1 g + m_1 R\alpha) = m_2 gR - m_2 R^2\alpha - m_1 gR - m_1 R^2\alpha$$

$$\Rightarrow I\alpha + m_2R^2\alpha + m_1R^2\alpha = m_2gR - m_1gR \Rightarrow \alpha\left(I + m_2R^2 + m_1R^2\right) = gR\left(m_2 - m_1\right)$$

$$\Rightarrow \alpha = \frac{gR\left(m_2 - m_1\right)}{I + m_2R^2 + m_1R^2} = \frac{gR\left(m_2 - m_1\right)}{I + R^2\left(m_2 + m_1\right)} = \frac{(9.81)(0.0385)(0.2-0.1)}{0.0137 + (0.0385)^2(0.2+0.1)} = 2.67016 \approx \boxed{2.67\frac{rad}{s^2}}$$

Compare to the measured angular acceleration:

$$\omega_i = 0;\ \Delta\theta = 2rev \times \frac{2\pi rad}{1rev} = 4\pi rad;\ \Delta t = 193 frames \times \frac{1\sec}{60 frames} = 3.21\overline{6}\sec;\ \alpha = ?$$

$$\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2 = \frac{1}{2}\alpha\Delta t^2 \Rightarrow \alpha = \frac{2\Delta\theta}{\Delta t^2} = \frac{(2)(4\pi)}{\left(3.21\overline{6}\right)^2} = 2.4290 \approx 2.43\frac{rad}{s^2}$$

$$E_r = \frac{O - A}{A} \times 100 = \frac{2.4290 - 2.67016}{2.67016} \times 100 = -9.0315 \approx -9.03\%$$

Now let's find the tensions:

$$F_{T_1} = m_1g + m_1a_1 = m_1\left(g + R\alpha\right) = (0.1)\left(9.81 + (0.0385)(2.67016)\right) = 0.991280 \approx \boxed{0.991N}$$

$$F_{T_2} = m_2g - m_2a_2 = m_2\left(g - R\alpha\right) = (0.2)\left(9.81 - (0.0385)(2.67016)\right) = 1.94144 \approx \boxed{1.94N}$$

Now, please notice that those two forces of tension are not equal in magnitude. Recall that is because our pulley has mass and therefore has a rotational inertia and therefore requires a net torque to angularly accelerated it. Therefore, in order to cause that net torque on the pulley, force of tension 1 and force of tension 2 cannot have the same magnitude. But if the pulley had negligible mass and therefore negligible rotational inertia, the equation we got from summing the net torques would actually show that the two forces of tension acting on the pulley would be the same.

$$RF_{T_2} - RF_{T_1} = I\alpha = (0)\alpha = 0 \Rightarrow RF_{T_2} = RF_{T_1} \Rightarrow F_{T_2} = F_{T_1}$$

I also want to point out two ways which students try to use to solve this problem which are both incorrect.

1) Sum the torques on the whole system all at once with the axis of rotation at the axle of the pulley. But realize, the rotational inertia and angular acceleration on the right hand side of the equation would then refer to everything in the system, including the two hanging masses. And hopefully you recognize that the hanging masses do not have rotational inertia nor do they have angular acceleration. So this equation is **incorrect**.

$$\sum_{\substack{\text{everything}\\ AoR\ @\ Axle}} \vec{\tau} = \vec{\tau}_{F_{g_2}} - \vec{\tau}_{F_{T_2}} + \vec{\tau}_{F_{T_2}} - \vec{\tau}_{F_{T_1}} + \vec{\tau}_{F_{T_1}} - \vec{\tau}_{F_{g_1}} = I\alpha$$

2) Sum the forces on the entire system in the positive direction. This time it is because the mass times acceleration on the right hand side of the equation would be for the entire system. Now, I do understand that the rim of the pulley does have the same magnitude tangential acceleration as the linear accelerations of the two masses, however, the pulley itself does not have a tangential acceleration because tangential acceleration depends on radius. The larger the radius the larger the tangential acceleration of whatever specific point on the pulley you are referring to. So, this equation is also **incorrect**.

$$\sum_{\substack{\text{everything}\\ +\ direction}} \vec{F} = \vec{F}_{g_2} - \vec{F}_{T_2} + \vec{F}_{T_2} - \vec{F}_{T_1} + \vec{F}_{T_1} - \vec{F}_{g_1} = ma\ \&\ a_t = r\alpha$$