Flipping Physics Lecture Notes:
Conservation of Angular Momentum Introduction and Demonstrations

We have already learned about conservation of linear momentum: $\sum \vec{p}_i = \sum \vec{p}_f$

As you can imagine, the equation for conservation of angular momentum is similar: $\sum \vec{L}_i = \sum \vec{L}_f$

We will talk about when angular momentum is conserved in a little bit, let's start however, with a simple demonstration of conservation of angular momentum.

If I am spinning on a stool holding two masses at arm's length straight out from my body and then pull in my arms, what will happen? From general knowledge of sports like figure skating, gymnastics, dance, and diving, you probably already know that my angular velocity will increase. It is important to know why. The equation for angular momentum of a rigid object with shape is: $\vec{L} = I\vec{\omega}$. Therefore, the equation for conservation of angular momentum in the example of me spinning on a stool is:

$$\sum \vec{L}_i = \sum \vec{L}_f \Rightarrow I_i\vec{\omega}_i = I_f\vec{\omega}_f$$

It is important to recognize that the axis of rotation for the angular momenta and rotational inertias in these equations is the vertical axis through the center of the stool.

The equation for the rotational inertia of a system of particles is: $I_{particles} = \sum_i m_i r_i^2$

In other words, bringing in my arms decreases the average distance the particles of the system are from the axis of rotation, which decreases the rotational inertia of the system, therefore, because angular momentum is conserved, the angular velocity of the system must increase.

$$r \downarrow \Rightarrow I \downarrow \Rightarrow \vec{\omega} \uparrow$$

Now let's talk about when angular momentum is conserved. As you recall linear momentum is conserved when the net force on the system equals zero.

$$\sum \vec{F}_{\substack{external \\ on\ system}} = 0 = \frac{\Delta \vec{p}_{system}}{\Delta t} \Rightarrow 0 \bullet \Delta t = \left(\frac{\Delta \vec{p}_{system}}{\Delta t}\right)\Delta t \Rightarrow 0 = \Delta \vec{p}_{system} = \vec{p}_{f\,system} - \vec{p}_{i\,system} \Rightarrow \vec{p}_{i\,system} = \vec{p}_{f\,system}$$

Angular momentum is conserved when the net external torque acting on the system equals zero.

$$\sum \vec{\tau}_{\substack{external \\ on\ system}} = 0 = \frac{\Delta \vec{L}_{system}}{\Delta t} \Rightarrow 0 \bullet \Delta t = \left(\frac{\Delta \vec{L}_{system}}{\Delta t}\right)\Delta t \Rightarrow 0 = \Delta \vec{L}_{system} = \vec{L}_{f\,system} - \vec{L}_{i\,system} \Rightarrow \vec{L}_{i\,system} = \vec{L}_{f\,system}$$

Hopefully you can see the similarities in the derivations. ☺

Returning back to me sitting on the stool. Notice that, about the vertical axis through the center of the stool, the net external torque acting on the system of me and the stool is zero, therefore angular momentum of the system will stay constant. The initial angular velocity of the system is zero, therefore, I can wave my arms around all I want, but doing so will not change the angular momentum of the system. However, if I push on something external to the system, I can cause a net torque on the system, angular momentum is no longer conserved, and I can increase my angular velocity.

Remember that angular momentum is a vector, therefore, when angular momentum is conserved, its direction is conserved as well. This is why a spinning top will maintain its vertical position. Its angular momentum, according to the right hand rule, will be vertical and therefore, as long as the top continues to spin, the angular momentum will be conserved and the top will stay vertical. However, a top which is not spinning, has no angular momentum, and will not stay vertical.

We can also apply this concept to a moving bicycle. The wheels of the bike, while the bike is moving, are spinning and have angular momentum. While you are moving forward, the direction of the angular momentum of the wheels will be to the left. Conservation of angular momentum will try to maintain the direction of the angular momentum of the wheels and therefore will help keep the bicycle vertical. If the bike is not moving, the wheels have no angular momentum and therefore do not help keep the bicycle vertical. Conservation of angular momentum is why it is easier to balance on a bike while it is moving.