Flipping Physics Lecture Notes:

Conservation of Charge Example Problems
https://www.flippingphysics.com/conservation-of-charge.html

Conservation of Charge: The total electric charge of an isolated system never changes.

What is an isolated system? We could start with the universe. In other words, the net electric charge of the universe never changes. Add up all the positive charges and subtract all the negative charges and you will always get the same number.

Or we could have a smaller isolated system, like the conductive metal pieces of an electroscope which are electrically isolated from the rest of the universe by the rubber and glass insulators. In other words, the net electric charge of the electroscope will remain constant, as long as it remains isolated.

Example Problem #1: Two charged, conducting objects collide and separate. Before colliding, the charges on the two objects are +3e and -6e. Which of the following are possible values for the final charges on the two objects? Choose all possible answers.

(a) +4e, -7e (b) +2e, -2e (c) -1.5e, -1.5e (d) -3.5e, +2.5e (e) +e, -4e

$q_{1i} = +3e;\ q_{2i} = -6e;\ q_{1f} = ?;\ q_{2f} = ?$ & $q_{total\ i} = q_{1i} + q_{2i} = +3e + (-6e) = -3e = q_{total\ f}$

a) $q_{1f} + q_{2f} = +4e + (-7e) = -3e = q_{total\ f}$ ✓ it works

b) $q_{1f} + q_{2f} = +2e + (-2e) = 0 \neq -3e = q_{total\ f}$ **does not work**

c) $q_{1f} + q_{2f} = -1.5e + (-1.5e) = -3e = q_{total\ f}$
But (c) **does not work** because you cannot have half an electron because charge is quantized!!

d) $q_{1f} + q_{2f} = -3.5e + 2.5e = -e = q_{total\ f}$ **does not work**
But (d) also does not work because you cannot have half an electron because charge is quantized!!

e) $q_{1f} + q_{2f} = +e + (-4e) = -3e = q_{total\ f}$ ✓ it works

Correct answers are (a) and (e) because those are the only two options which have a total final charge equal to the total initial charge and are integer multiples of the fundamental charge e.

Example Problem #2: Two identical, conducting spheres are held using insulating gloves a distance x apart. Initially the charges on each sphere are +3.0 pC and +6.0 pC. The two spheres are touched together and returned to the same distance x apart. You may assume x is the distance between their centers of charge.

(a) What is the final charge on each sphere?
(b) Is the final electric force between the two spheres increased, decreased, or the same when compared to the initial electric force?

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$$q_{1i} = +3.0\,pC;\ q_{2i} = +6.0\,pC;\ r_i = r_f = x;\ \text{Part(a)}:\ q_{1f} = ?;\ q_{2f} = ?;\ \text{Part(b)}:\ F_{ef}\ ?\ F_{ei}$$

$$q_{total\ f} = q_{total\ i} = q_{1i} + q_{2i} = +3\,pC + 6\,pC = +9\,pC$$

Because the two spheres are identical, after touching, the spheres will have equal charge.

$$q_{1f} = q_{2f} = q_f \Rightarrow q_{total\ f} = q_{1f} + q_{2f} = 2q_f = 9\,pc \Rightarrow q_f = 4.5\,pC$$

(a) Both charges end with 4.5 pC of charge.

This is 4.5 picocoulombs of charge or 4.5 x 10^{-9} C which an object is physically able to have. Because then it will have:

$$q_f = n_f e \Rightarrow n_f = \frac{q_f}{e} = \frac{4.5\times10^{-9}C}{1.60\times10^{-19}\dfrac{C}{\text{charge carrier}}} = 2.8125\times10^{10} \approx 2.8\times10^{10}\,\text{excess protons}$$

Imagine that. 28 billion more protons than electrons on each sphere. Each sphere will have a heck of a lot more total protons and electrons, however, it has a deficit of 28 billion electrons and therefore has a net charge of 4.5 pC.

And now part (b): The two spheres have like charges, so they are repelled from one another with an electric force with a magnitude of: $F_e = \dfrac{kq_1q_2}{r^2}$ Therefore …

$$F_{ei} = \frac{kq_{1i}q_{2i}}{r_i^{\,2}} = \frac{k\left(3\times10^{-9}\right)\left(6\times10^{-9}\right)}{x^2} = 1.8\times10^{-17}\frac{k}{x^2}$$

$$F_{ef} = \frac{kq_{1f}q_{2f}}{r_f^{\,2}} = \frac{k\left(4.5\times10^{-9}\right)\left(4.5\times10^{-9}\right)}{x^2} = 2.025\times10^{-17}\frac{k}{x^2} \approx 2.0\times10^{-17}\frac{k}{x^2}$$

k is the Coulomb Constant and has a constant value: $k = 8.99\times10^{9}\dfrac{N\bullet m^2}{C^2}$

x is also a constant. Therefore: $F_{ef} \approx 2.0\times10^{-17}\dfrac{k}{x^2} > 1.8\times10^{-17}\dfrac{k}{x^2} = F_{ei}$

The final electric force is *greater than* the initial electric force.