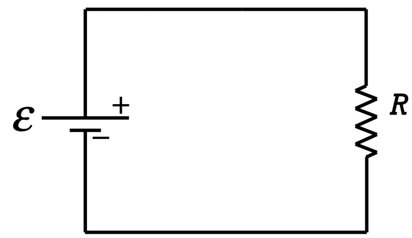
Flipping Physics Lecture Notes:

Kirchhoff's Rules of Electrical Circuits

https://www.flippingphysics.com/kirchhoff.html

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0$$



Kirchhoff's Two Rules for circuits are very basic rules which are used to understand circuits. Let's start with Kirchhoff's Loop Rule which states that the net electric potential difference around a closed loop equals zero.

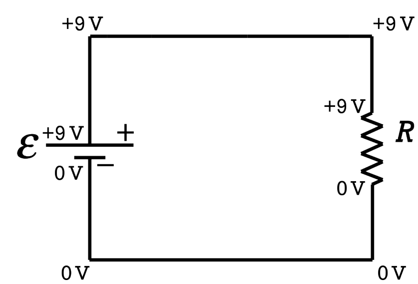
The Loop Rule is essentially conservation of electric potential energy in a circuit. Because electric potential difference equals change in electric potential energy per unit charge, the net change in electric potential energy in a closed loop then equals zero.

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0 \;\&\; \Delta V = \frac{\Delta U_e}{q} \Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \frac{\Delta U_e}{q} = 0 \Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \Delta U_e = 0$$

Using a gravitational potential energy analogy here, this is like saying, if you drop a mass off a wall, then pick up the mass and return it to its original location, the change in gravitational potential energy of that mass equals zero. We know this to be true because the mass returns back to the same height as where it started, so the mass will have the same gravitational potential energy at the end as it did at the beginning, no matter where we place the horizontal zero line.

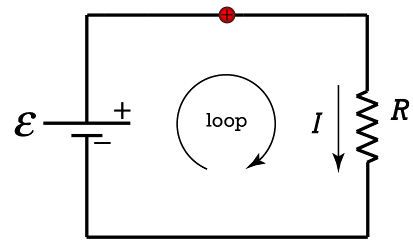
Going back to electric potential energy, this means, after a charge goes through one full, closed loop around a circuit, the electric potential energy of the charge will return back to its original value. But because we are using electric potential, we are really talking about the electric potential energy per unit charge at each location.

Let's say we have a 9-volt battery. That means we know the electric potential difference across the battery equals 9 volts. As we go from the negative to the positive terminals of the battery, the electric potential will go up. Technically we do not know the electric potential at any point, only the *difference* in the electric potential, however, it is customary to assume the minimum electric potential is zero. That means we are assuming the negative terminal of the battery is at zero volts and the positive terminal of the battery is at positive 9 volts.



Because ideal wires have zero resistance, that means the electric potential in the upper left corner must also be 9 volts, the electric potential in the upper right corner equals 9 volts, and the electric potential at the top of the resistor is 9 volts. Also, the electric potential in the lower left corner must be the same as the negative terminal of the battery, so electric potential in the lower left corner equals 0 volts. Therefore, electric potential in the lower right corner is 0 volts, and the electric potential at the bottom of the resistor equals 0 volts. This means the electric potential difference across the resistor also has a magnitude of 9 volts. In other words, in this circuit with two circuit elements, the two elements, the battery and the resistor, both have the same magnitude electric potential difference.

In a previous lesson we determined that a positive charge in the circuit would be repelled from the positive terminal of the battery and attracted to the negative terminal of the battery, therefore the current in this circuit is clockwise. This means the current is down through the resistor.

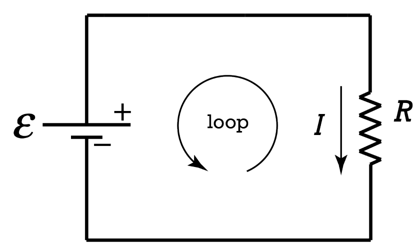


There is only one closed loop in our present circuit, so it might not seem obvious that we need to do this, however, we need to define a loop direction. Often the loop direction is the same as the direction which goes from the negative terminal to the positive terminal of the battery and through the battery, therefore, our loop direction for this circuit is clockwise.

This means as we go in the direction of the loop across the battery, the electric potential goes up because we go from the negative to the positive terminal of the battery. Therefore, when we sum the electric potential differences in our Kirchhoff's loop equation, the electric potential difference across the battery is positive. When we go in the direction of the loop across the resistor, as we illustrated before, the electric potential goes down. Therefore, in our loop equation, the electric potential difference across the resistor is negative. We know the electric potential difference across the battery equals the electromotive force or the emf of the battery. And the electric potential difference across the resistor equals current times resistance. Therefore, we can determine the current in the circuit in terms of the emf of the battery and the resistance of the resistor.

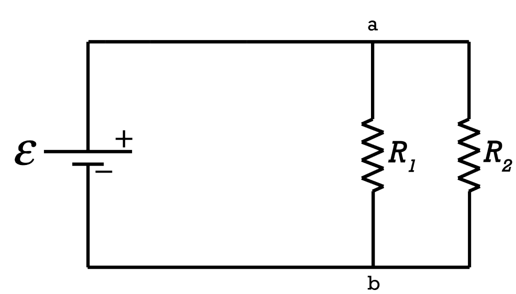
$$\Delta V_{Battery} = \varepsilon \ \& \ \Delta V_{Resistor} = IR$$

$$\Rightarrow \sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = \Delta V_{battery} - \Delta V_{Resistor} = 0 = \varepsilon - IR \Rightarrow \varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R}$$



If we had chosen counterclockwise as the loop direction, all of our electric potential differences in Kirchhoff's Loop Rule would have been reversed. Because the loop direction goes from the positive to the negative terminals of the battery, the electric potential difference across the battery is negative, because the electric potential is going down. Because the loop direction through the resistor is opposite the direction of the current direction we defined through the resistor, the electric potential goes up through the resistor and the electric potential difference the across resistor is positive.

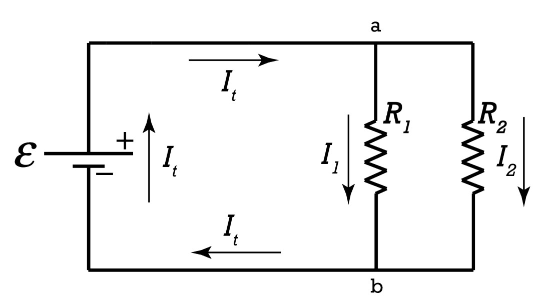
$$\Rightarrow \sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = -\Delta V_{battery} + \Delta V_{Resistor} = 0 = -\varepsilon + IR \Rightarrow \varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R}$$



Realize, we get the same result for the current in the circuit regardless of which loop direction we choose. If we had chosen an incorrect direction for current, the current ends up being negative, which tells you that you chose the incorrect current direction.

Now let's add a resistor to the circuit and talk about Kirchhoff's Junction Rule which is the result of conservation of charge in the circuit. The rule is sum of the currents entering a junction must equal the sum of the currents leaving a junction, which is conservation of charge:

$$\sum I_{in} = \sum I_{out}$$



Junctions are locations in circuits where at least three circuit paths meet. That means in our circuit we have two junctions which are labelled *a* and *b*. Therefore, the current going into both of those junctions equals the current coming out of those junctions. This means we need to define current directions. We do this the same way we did before, we place a positive test charge in the circuit and see which direction the Law of Charges defines electric force direction on the charge. This means current will go to the right through the top wire, to the left through the bottom wire, and down through both resistors. Let's label those currents as current 1 through resistor 1, current 2 through resistor 2, and current t through the battery because it is the current through the terminals of the battery.

Kirchhoff's Junction Rule equations for this circuit are for:

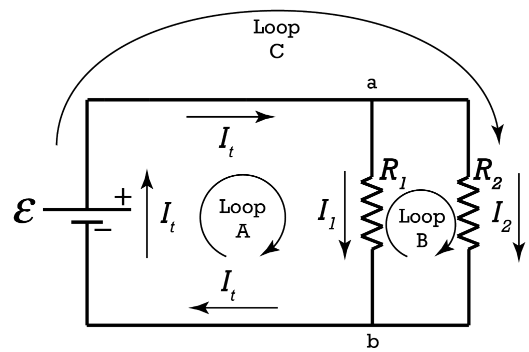
junction a: $I_t = I_1 + I_2$

junction b: $I_t = I_1 + I_2$

Yes, these two equations are actually the same.

But how do we know *a* and *b* are junctions and the four exterior "corners" of the circuit are not junctions? I know it may seem obvious because there are not at least three circuit paths at any of those locations, however, this is a simple circuit. Again, we return back to placing a positive test charge on the wire. Notice that a charge which approaches point a could go in the wire leading to resistor 1 or in the wire leading to resistor 2. Because junctions are defined as having three circuit paths, any time a charge comes to a fork in the wire, the charge could go down either wire, that makes it a junction. When a charge enters a corner, there is no other choice but to continue along the same wire, therefore none of the corners are junctions.

Let's identify the loops in this second circuit and determine their Kirchhoff's Loop Rule equations. We can define the first loop as the same as the previous circuit, but let's call it loop A with a clockwise direction. There is another loop that contains resistor 1 and resistor 2. Let's call that loop B and also have that be clockwise. Lastly there is a loop all the way around the outside; It includes the battery and resistor 2. Let's call that loop C and have it also be clockwise.



Kirchhoff's Loop Rule equations look like this:

$$\sum_{Loop\ A} \Delta V = \Delta V_t - \Delta V_{R_1} = \varepsilon - I_1 R_1 = 0 \Rightarrow \varepsilon = I_1 R_1 \Rightarrow I_1 = \frac{\varepsilon}{R_1}$$

$$\sum_{Loop\ C} \Delta V = \Delta V_t - \Delta V_{R_2} = \varepsilon - I_2 R_2 = 0 \Rightarrow \varepsilon = I_2 R_2 \Rightarrow I_2 = \frac{\varepsilon}{R_2}$$

$$\sum_{Loop\ B} \Delta V = \Delta V_{R_1} - \Delta V_{R_2} = I_1 R_1 - I_2 R_2 = 0 \Rightarrow I_1 R_1 = I_2 R_2$$

But notice the third equation is actually just a combination of the previous two: $\varepsilon = I_1 R = I_2 R_2$