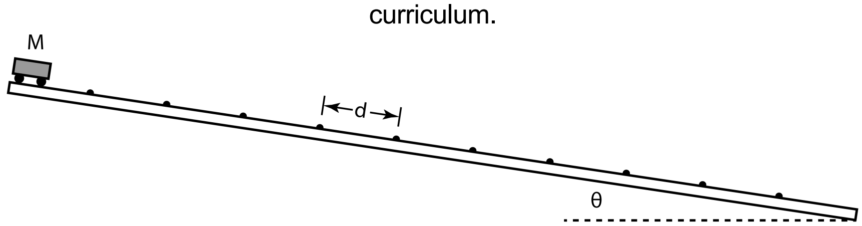
Flipping Physics Lecture Notes:

2016 #3 Free Response Question - AP Physics 1 - Exam Solution

http://www.flippingphysics.com/ap1-2016-frq3.html

This Quantitative/Qualitative Translation question also works as a part of the AP Physics C: Mechanics curriculum.



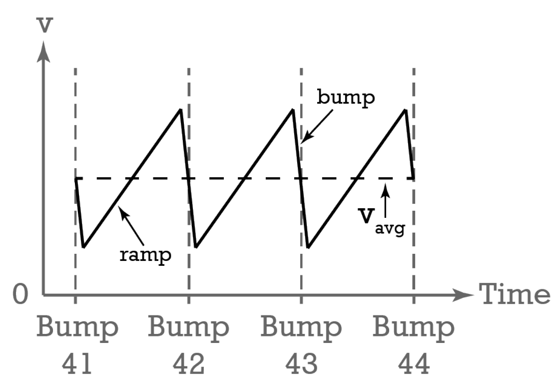
Note: Figure not drawn to scale.

A long track, inclined at an angle θ to the horizontal, has small speed bumps on it. The bumps are evenly spaced a distance d apart, as shown in the figure above. The track is actually much longer than shown, with over 100 bumps. A cart of mass M is released from rest at the top of the track. A student notices that after reaching the 40th bump the cart's average speed between successive bumps no longer increases, reaching a maximum value v_{avg}. This means the time interval taken to move from one bump to the next bump becomes constant.

(a) Consider the cart's motion between bump 41 and bump 44.

i. In the figure below, sketch a graph of the cart's velocity v as a function of time from the moment it reaches bump 41 until the moment it reaches bump 44.

ii. Over the same time interval, draw a dashed horizontal line at $v = v_{avg}$. Label this line "v_{avg}".



We know the cart reaches a maximum speed value, v_{avg}, down the incline. In the absence of bumps, the cart would have a constant acceleration down the incline. Therefore, between bumps, the cart will have a constant positive acceleration down the incline, a_{ramp}. A constant positive acceleration on a velocity as a function of time graph is a straight line with a positive slope. When the cart is going over a bump, the cart must have a large acceleration up the incline, therefore, this acceleration will be negative, a_{bump}. We will assume this negative acceleration caused by each bump is constant. A large, constant negative acceleration on a velocity as a function of time graph is a straight line with a large negative slope. Therefore, as the cart goes over each bump, there will be straight line with a large negative slope and, as the cart goes between each bump, there will be a straight line with a positive slope of a smaller magnitude. Add a dotted, horizontal line in the middle of the velocity graph and label it velocity average.

Notes about grading:

- *1 point is earned for this graph for having a "minimum positive value that is the same for each bump".*
- *1 point is earned for having the "same maximum value in each cycle that occurs near the bump times".*
    - *In other words, the grader must be able to see, from your drawing, that the maximum values are equal and all the minimum values are equal. If you are not careful, you could lose points there. And if you are struggling with the drawing, you can always add a note indicating what you were trying to draw.*
- *1 point is earned for your v_{avg} line if it is "horizontal and consistent with the graph drawn, even if that graph is wrong". In other words, even if you got the other parts of the drawing completely wrong, you can still earn a point for drawing a v_{avg} line which is "horizontal and consistent with the graph drawn"!*
    - *I cannot stress this enough. Answer every question! Even if you think every other part of your answer is completely wrong.*

(b) Suppose the distance between the bumps is increased but everything else stays the same.

Is the maximum speed of the cart now greater than, less than, or the same as it was with the bumps closer together?

_X_ Greater than ____ Less than ____ The same as

Briefly explain your reasoning.

The only thing which has changed is the distance between bumps which is the distance during which the cart accelerates. The magnitudes of both the a_{ramp} and a_{bump} are unchanged. Because there is a larger distance for the cart to accelerate between the bumps, the cart will be able to accelerate to a larger final velocity before each bump, therefore, the average maximum speed of the cart will now be ***greater than*** *it was before.*

Grading note: "No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation." In other words, when they ask you to "briefly explain your reasoning", they mean it. This is why I do not give any points for a correct answer with an incorrect solution or no solution at all. Because, the solution shows you understand what you are doing. The answer is just, the answer.

(c) With the bumps returned to the original spacing, the track is tilted to a greater ramp angle θ. Is the maximum speed of the cart greater than, less than, or the same as it was when the ramp angle was smaller?

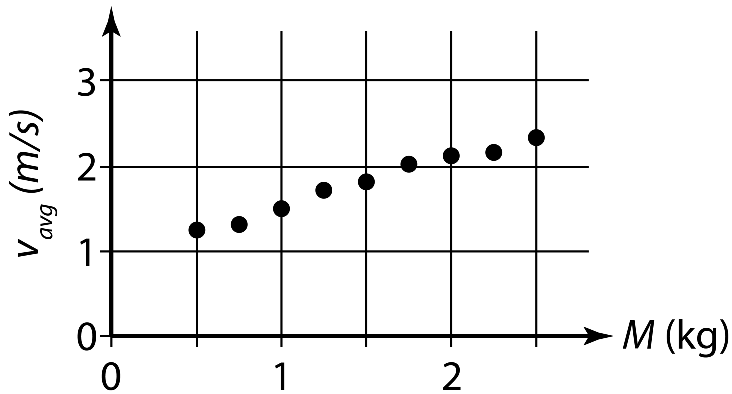
_X_ Greater than ____ Less than ____ The same as

Briefly explain your reasoning.

Reset to the original ramp and change the ramp angle θ. This will change the magnitude of the component of the force of gravity which is parallel to and down the ramp. Because the angle is increased, the force of gravity parallel will be increased, which will increase the acceleration between the bumps, which will lead to a larger final velocity before each bump, therefore, the average maximum speed of the cart will now be ***greater than*** *it was before.*

(d) Before deriving an equation for a quantity such as v_{avg}, it can be useful to come up with an equation that is intuitively expected to be true. That way, the derivation can be checked later to see if it makes sense physically. A student comes up with the following equation for the cart's maximum average speed: $v_{avg} = C\dfrac{Mg\sin\theta}{d}$ where C is a positive constant.

i. To test the equation, the student rolls a cart down the long track with speed bumps many times in front of a motion detector. The student varies the mass M of the cart with each trial but keeps everything else the same. The graph shown below is the student's plot of the data for v_{avg} as a function of M.



Are these data consistent with the student's equation?

____ Yes __X__ No

Briefly explain your reasoning.

The equation provided by the student predicts that, as the mass of the cart approaches zero, the average speed of the cart would approach zero, however, a best-fit line of the data has nonzero y-intercept.

Also, the equation provided by the student predicts that increasing the mass of the cart by a factor of two should increase the v_{avg} by a factor of two. The v_{avg} at 1 kg is roughly 1.5 m/s. The v_{avg} at 2 kg should then be roughly 3 m/s, however, it is only roughly 2.1 m/s.

ii. Another student suggests that whether or not the data above are consistent with the equation, the equation could be incorrect for other reasons. Does the equation make physical sense?

____ Yes __X__ No

Briefly explain your reasoning.

The equation provided by the student predicts that, as the distance between the bumps increases, the v_{avg} should decrease. We have already shown, in part (b), the reverse is true; as d increases, the average speed also increases. No, the equation does not make physical sense.

Grading note: "If 'yes' is selected, one point [out of two] can be earned for indicating that an increase in the angle increases v_{avg}." In other words, if you get the answer wrong, you can still earn a point for correctly arguing that v_{avg} increases with increasing ramp angle. Which reiterates my point that, even if you think you are wrong, answer every single question!! Because you can have the wrong answer and still earn points with a correct explanation.