Flipping Physics Lecture Notes:

2018 #3 Free Response Question - AP Physics 1 - Exam Solution
http://www.flippingphysics.com/ap1-2018-frq3.html

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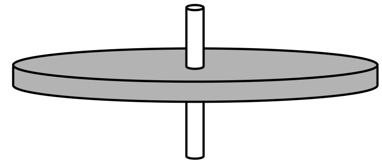
This Quantitative/Qualitative Translation question also works as a part of the AP Physics C: Mechanics curriculum.

The disk shown spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

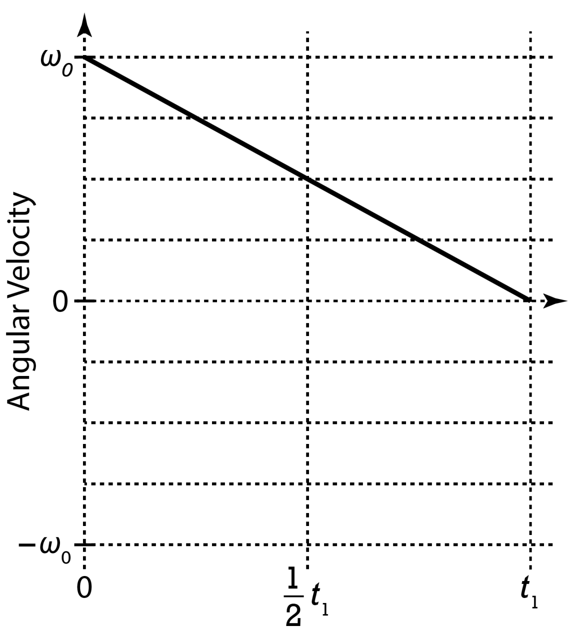


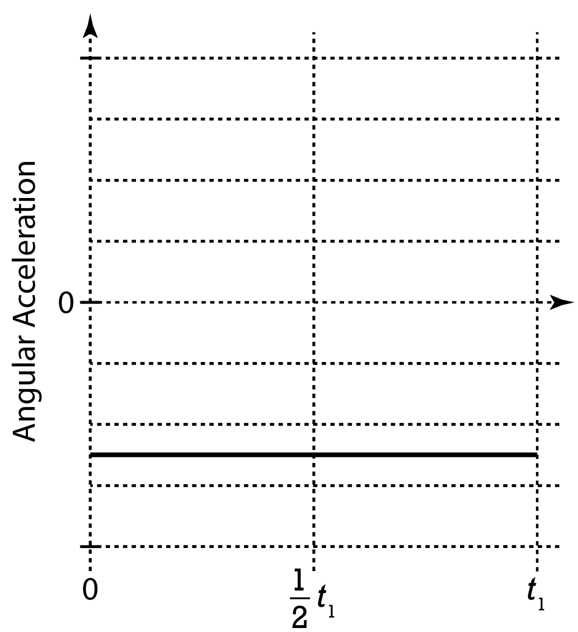
(a) At time t = 0 the disk has an initial counterclockwise (positive) angular velocity ω_0. The disk later comes to rest at time $t = t_1$.

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from t = 0 until the disk comes to rest at time $t = t_1$.

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from t = 0 until the disk comes to rest at time $t = t_1$.

Friction is the only force causing a torque on the disk about its center of mass. Therefore, friction causes a constant net torque on the disk which causes the disk to have a constant clockwise (negative) angular acceleration. A constant negative angular acceleration on an angular velocity as a function of time graph is a line with a constant negative slope. The initial angular velocity is ω_0 and the final angular velocity is zero at time $t = t_1$. So, the angular velocity as a function of time graph is a straight line with a negative slope starting at ω_0 and ending at 0. A constant angular acceleration on an angular acceleration as a function of time graph is a horizontal line. Because the angular acceleration is negative, the horizontal line is below the x-axis.





Notes about grading. Part (a) is worth 4 points out of 12 for this question. One third of the points are for drawing two lines, one on each graph. No explanation is necessary at all. This should highlight the importance of the graphs they ask you to draw. Please draw carefully!

(b) The magnitude of the frictional torque exerted on the disk is τ_0. Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

$$\sum \tau = -\tau_0 = I\alpha \;\&\; \alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{0 - \omega_0}{t_1 - 0} = -\frac{\omega_0}{t_1}$$

$$\Rightarrow I = -\frac{\tau_0}{\alpha} = -\frac{\tau_0}{-\frac{\omega_0}{t_1}} = \frac{\tau_0 t_1}{\omega_0}$$

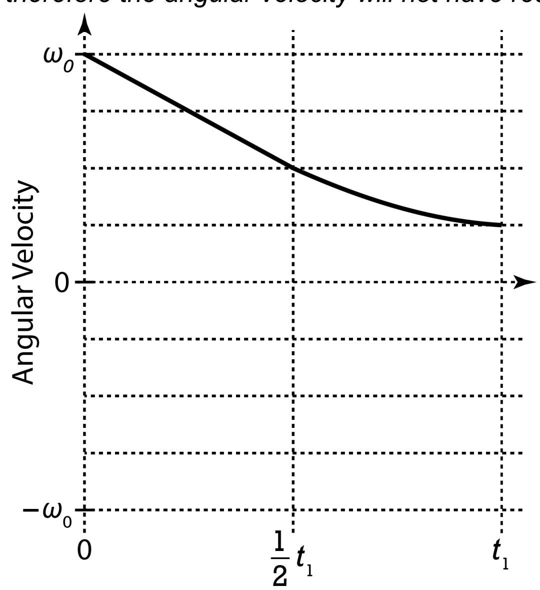
Part (b) is only worth 3 points. In other words, the graphs are worth more than the equation derivation. Remember when I said you need to be careful drawing your graphs?

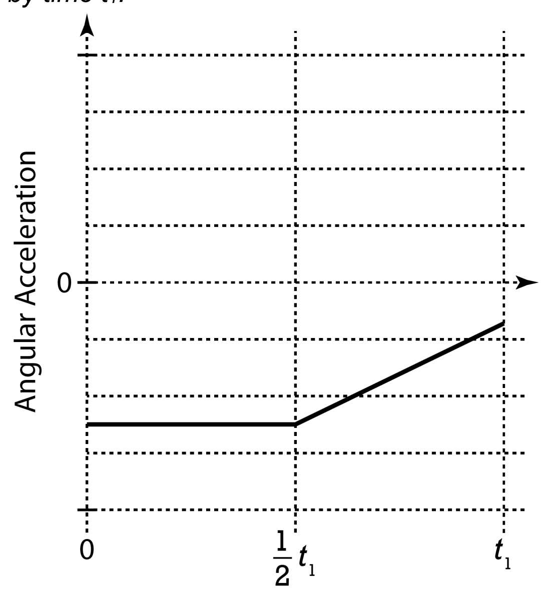
(c) In another experiment, the disk again has an initial positive angular velocity ω_0 at time t = 0. At time t = ½ t_1, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from t = 0 to t = t_1, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from t = 0 to t = t_1.

First off, realize nothing has changed from time t = 0 to time t = ½ t_1, so the first half of both graphs should be the same as they were in part (a). For the second half of each graph, the oil reduces the force of friction which reduces the net torque which reduces the angular acceleration of the wheel. This means the second half of the angular acceleration graph should show a decreasing angular acceleration, in other words, the line should move towards the horizontal time axis. For the second half of the angular velocity graph, the angular acceleration is no longer constant, so the line is no longer straight, but rather is curved. The slope of an angular velocity as a function of time graph is angular acceleration, so the slope of the angular velocity graph from ½ t_1 to t_1 should slowly increase in value or get flatter and the angular velocity at time t_1 needs to be positive because the angular acceleration is less than it was in part (a) and therefore the angular velocity will not have reached zero by time t_1.





More about grading. 2 more graphs worth a total of 4 more points. 8 total points out of 12 for question #3 all for graphs with no explanation. If you did not believe me before, believe me now. Be careful when drawing your graphs!!

(d) The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant (C_1 or C_2) with appropriate units.

(1) $\tau_1(t) = C_1\left(t - \frac{1}{2}t_1\right)$ (for $t > \frac{1}{2}t_1$)

(2) $\tau_2(t) = \dfrac{C_2}{\left(t + \frac{1}{2}t_1\right)}$ (for $t > \frac{1}{2}t_1$)

Which equation better mathematically models this experiment?

____ Equation (1) _X_ Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

We can determine the torque for each equation at time $t = \frac{1}{2}t_1$ and $t = t_1$.

$$\tau_1(t) = C_1\left(t - \frac{1}{2}t_1\right) \Rightarrow \tau_1\left(\frac{1}{2}t_1\right) = C_1\left(\frac{1}{2}t_1 - \frac{1}{2}t_1\right) = 0 \;\&\; \tau_1(t_1) = C_1\left(t_1 - \frac{1}{2}t_1\right) = C_1\left(\frac{1}{2}t_1\right)$$

$$\tau_2(t) = \frac{C_2}{\left(t + \frac{1}{2}t_1\right)} \Rightarrow \tau_2\left(\frac{1}{2}t_1\right) = \frac{C_2}{\left(\frac{1}{2}t_1 + \frac{1}{2}t_1\right)} = \frac{C_2}{t_1} \;\&\; \tau_2(t_1) = \frac{C_2}{\left(t_1 + \frac{1}{2}t_1\right)} = \frac{C_2}{\frac{3}{2}t_1} = \frac{2}{3}\left(\frac{C_2}{t_1}\right)$$

The torque for equation 1 does not make sense because it would be zero at time $\frac{1}{2}t_1$ and gets larger from there. The magnitude of the torque should start out at a positive value at time $\frac{1}{2}t_1$ and decrease from there, which is what equation 2 does. So, equation 2 is the one which better mathematically models this experiment.