Flipping Physics Lecture Notes:

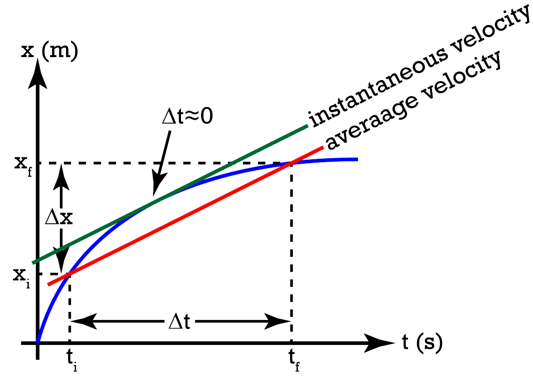
Derivative Introduction

http://www.flippingphysics.com/derivative.html

The velocity of an object in the x direction is defined as:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

In other words, the velocity of an object equals the limit of the average velocity of an object as the change in time for that average velocity approaches zero.



On a graph of position as a function of time I have displayed, in blue, the motion of an object. The red line illustrates the change in position over change in time of an object or the object's **average velocity** over that change in time. If we decrease that change in time until it is infinitesimally small, we get the line in green which represents the **instantaneous velocity** of the object at that specific point in time and at that specific location. This limit is called the *derivative* and it is written like this:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

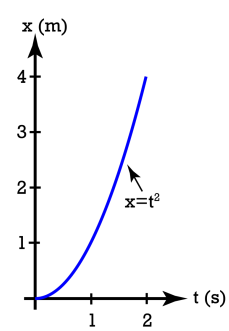
dx over dt means: The derivative of x with respect to t or the derivative of position with respect to time.

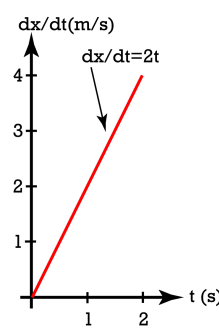
The definition of the derivative is the rate of change of a function with respect to a variable.
For our purposes, the derivative represents the slope of the tangent line of a curve. As you can see in our graph, the slope of the blue motion curve at a specific point is represented in green, and the derivative of the curve at that point is the slope of the green line. When looking at it as the derivative of position as a function of time, the derivative is instantaneous velocity. Please realize the derivative could also be:

$$\frac{dv}{dt} \text{ or } \frac{dp}{dt} \text{ or } \frac{dL}{dt} \text{ or } \frac{dW}{dt} \text{ or } \frac{dx}{dy}$$

Usually we are interested in the rate of change of a function with respect to time, however, that is not always the case.
Now let's look at how to take derivatives. The derivative rule we start with is the derivative of a power function:

$$\frac{d}{dt}\left(At^n\right) = nAt^{(n-1)}$$





And here is an example using the rule:

$$x(t) = t^2 \Rightarrow \frac{dx}{dt} = \frac{d}{dt}\left(t^2\right) = 2t^{(2-1)} = 2t^1 = 2t$$

Remember the derivative represents the slope of a function. In other words the slope of the function x(t) = t^2 is 2t.

Another example:

$$y(t) = 4t^3 \Rightarrow \frac{dy}{dt} = (3)(4)t^2 = 12t^2$$

And one more:

$$x\left(y\right)=6y^{4}-5y+3\Rightarrow\frac{dx}{dy}=24y^{3}-5$$

Let’s talk about why the derivative of a constant like 3 equals zero. Remember the derivative represents the slope of the function and a function which has a constant value has a slope of zero.