Flipping Physics Lecture Notes:

Uniformly Accelerated Motion Equations and the Derivative
http://www.flippingphysics.com/uam-derivative.html

Here are the four Uniformly Accelerated Motion equations we have used so far:

$$v_f = v_i + a\Delta t;\ \Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2;\ v_f^{\ 2} = v_i^{\ 2} + 2a\Delta x;\ \Delta x = \frac{1}{2}\left(v_f + v_i\right)\Delta t$$

Typically, in a calculus-based class we also use different forms of these equations.

Let's start with the first equation: $v = v_0 + at$

Let's highlight the differences here starting with t instead of Δt.

- $\Delta t = t_f - t_i = t - 0 = t$
  - In other words, using t instead of Δt assumes the initial time is zero and labels the final time as just t.
- The, a, for constant acceleration is the same.
- Instead of velocity initial, v_i, we have v_0 or v "naught". The "naught" here is the digit zero. It means the velocity at time zero, hence v "naught". The word "naught" means zero.
- And just like we use t instead of t_f, we have v instead of v_f.

The second equation is: $x = x_0 + v_0 t + \frac{1}{2}at^2$

- Acceleration, a, is again the same.
- We have the same differences: v_0 instead of v_i, t instead of Δt.
- And now we have x_0 instead of x_i and x instead of x_f.
  - $x = x_0 + v_0 t + \frac{1}{2}at^2 \Rightarrow x - x_0 = v_0 t + \frac{1}{2}at^2 \Rightarrow \Delta x = v_0 t + \frac{1}{2}at^2$

The third and fourth equations are: $v^2 = v_0^{\ 2}t + 2a\left(x - x_0\right)$ & $x - x_0 = \frac{1}{2}\left(v + v_0\right)t$

The fourth equation is not on the equation sheet provided by The CollegeBoard, however, you are welcome to use it.

Let's see what happens when we take the derivative of the second equation with respect to time:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = \frac{d}{dt}\left(x_0\right) + \frac{d}{dt}\left(v_0 t\right) + \frac{d}{dt}\left(\frac{1}{2}at^2\right) = 0 + v_0 + 2\left(\frac{1}{2}at^1\right) = v_0 + at$$

$$\Rightarrow v = v_0 + at$$

Taking the derivative of the second uniformly accelerated motion equation with respect to time gives us the first UAM equation. This is why we have these new expressions for the uniformly accelerated motion equations. Note: This equation tells the instantaneous velocity of an object equals the velocity of the object at time zero plus the acceleration of the object times the time at which the instantaneous velocity occurs.

Using calculus, we begin to see relationships form between equations which, using only algebra, were more difficult to see before. When we get to integrals, we will be able to derive all of the UAM equations starting with the definition of acceleration.