Flipping Physics Lecture Notes:

Demonstration of Position, Velocity and Acceleration using the Derivative
http://www.flippingphysics.com/position-velocity-acceleration-derivative.html

The position as a function of time of an object is given by the equation $x(t)=\left(-0.689t^2+1.27t+1.20\right)m$. Determine the object's …

a) Initial position.
b) Velocity function.
c) Acceleration function.
d) Position at 1.50 seconds.
e) Velocity at 1.50 seconds.
f) Acceleration at 1.50 seconds.

a) $x(0)=-0.689(0)^2+1.27(0)+1.2=1.20m$

b) $v(t)=\frac{dx}{dt}=\frac{d}{dt}\left(-0.689t^2+1.27t+1.20\right)=-1.378t+1.27\approx\left(-1.38t+1.27\right)\frac{m}{s}$

c) $a(t)=\frac{dv}{dt}=\frac{d}{dt}\left(-1.378t+1.27\right)=-1.378\approx-1.38\frac{m}{s^2}$

This object is in uniformly accelerated motion with a constant acceleration of -1.38 m/s^2.

A quick detour to show we should have known that from the position equation. Let's determine the acceleration of this position equation:

$$x(t)=\left(7.00t^3-4.00\right)m\Rightarrow v=\frac{dx}{dt}=21.0t^2\frac{m}{s}\Rightarrow a=\frac{dv}{dt}=42.0t\frac{m}{s^2}$$

Notice this acceleration is ***not*** constant, therefore this is ***not*** uniformly accelerated motion.
If the position equation has an exponent larger than 2, it is ***not*** uniformly accelerated motion.
Therefore, we know from the original position equation that the object is in uniformly accelerated motion because the highest exponent is a 2.

Acceleration is also the second derivative of position as a function of time: $a(t)=\frac{dv}{dt}=\frac{d}{dt}\left(\frac{dx}{dt}\right)=\frac{d^2x}{dt^2}$

The velocity equation is one of the uniformly accelerated motion equations:

$$v=v_0+at$$

Velocity initial equals 1.27 m/s and acceleration equals -1.38 m/s^2.

The position equation is also one of the uniformly accelerated motion equations: $x=x_0+v_0t+\frac{1}{2}at^2$
Initial position equals 1.20 m, initial velocity equals 1.27 m/s, and 1/2a = -0.689 so a equals -1.38 m/s^2.

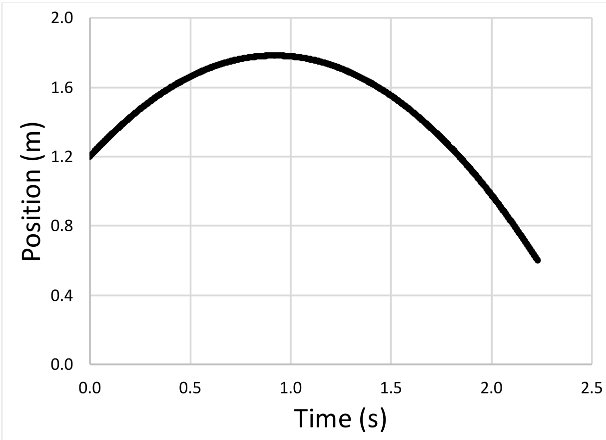
d) $x(1.5)=-0.689(1.5)^2+1.27(1.5)+1.2=-1.55035+1.905+1.2=1.55475\approx1.55m$

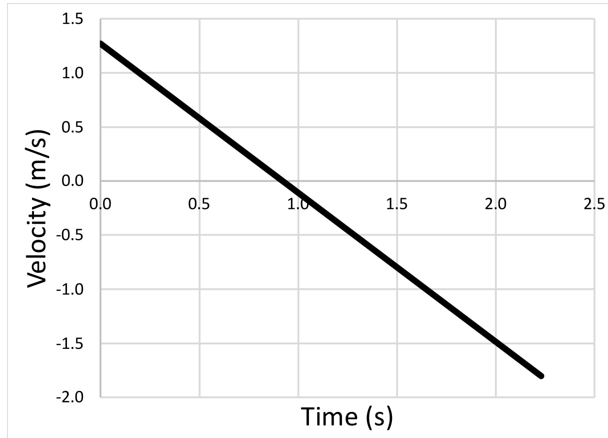
e) $v(1.5)=-1.378(1.5)+1.27=-0.797\frac{m}{s}$

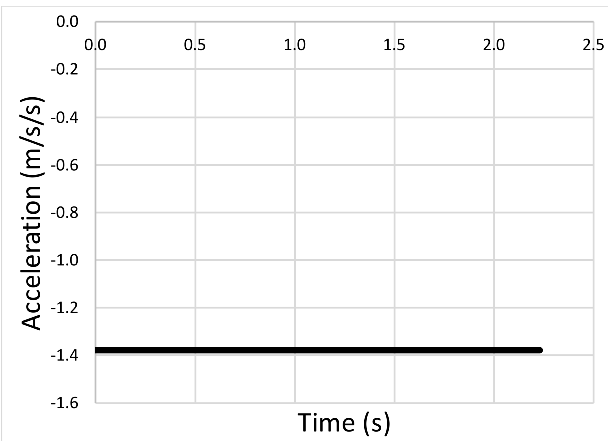
f) $a(1.5)=-1.378\approx-1.38\frac{m}{s^2}$

This equation is for a real event. This is an equation for a cart rolling up and then down an incline.

Let’s also look at graphs of these equations which describe the motion of the cart on the incline.







You can see everything we just talked about in these graphs:

- Position initial equals 1.20 m.
- Position final after 1.50 seconds equals 1.55 m.
- Initial velocity equals 1.27 m/s.
- Final velocity after 1.50 seconds equals -0.797 m/s.
- Acceleration is constant at -1.38 m/s^2.

And lastly, notice the point at with the velocity graph crosses the time axis is the time at which the cart has its maximum position. When the derivative of a function equals zero, the function has a local maximum, a local minimum, or a turning point. Let’s solve for that time.

$$v(t)=\frac{dx}{dt}=-1.378t+1.27=0\Rightarrow 1.27=1.378t\Rightarrow t=\frac{1.27}{1.378}=0.921626\approx 0.922\text{sec}$$

So, at 0.922 seconds, the cart is at its maximum position, which we can calculate:

$$x(0.921626)=-0.689(0.921626)^2+1.27(0.921626)+1.2=1.78523\approx 1.79m$$