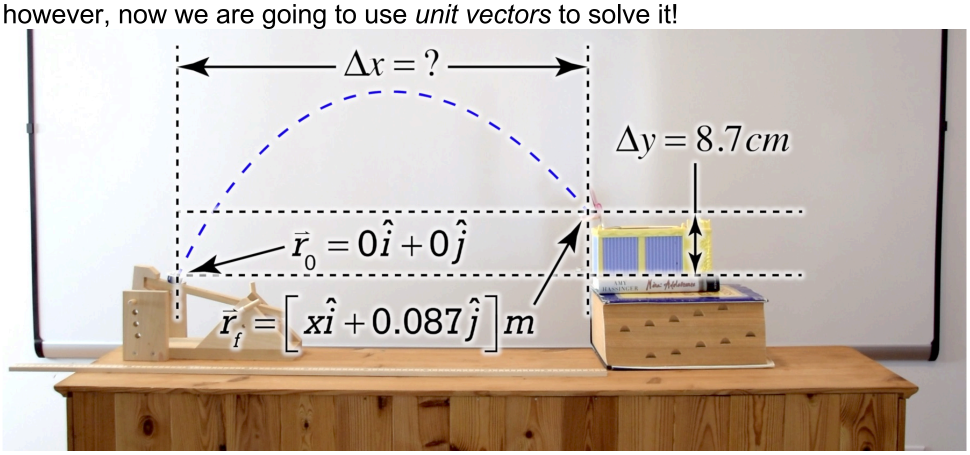
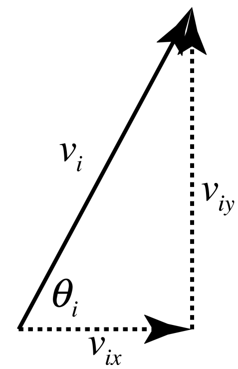
Flipping Physics Lecture Notes:
Nerd-A-Pult #2 using Unit Vectors
http://www.flippingphysics.com/nerd-a-pult-2-unit-vectors.html

A ball is launched from the Nerd-A-Pult with an initial speed of 3.25 m/s at an angle of 61.7° above the horizontal. If the basket is 8.7 cm above the ball vertically, where should the basket be placed horizontally relative to the ball so the ball lands in the basket?

We have already done this problem, https://www.flippingphysics.com/another-projectile-motion.html, however, now we are going to use *unit vectors* to solve it!





Let's identify the initial position of the ball as zero: $\vec{r}_0 = 0\hat{i} + 0\hat{j}$

The final position of the ball is 8.7 cm or 0.087 m in the y-direction and an unknown distance, x, in the x-direction: $\vec{r}_f = \left[x\hat{i} + 0.087\hat{j}\right]m$

We can use sine and cosine to determine the initial velocity of the ball in the x and y-directions:

$$\vec{v}_0 = \vec{v}_{0x}\hat{i} + \vec{v}_{0y}\hat{j} = \left(\vec{v}_0\cos\theta\right)\hat{i} + \left(\vec{v}_0\sin\theta\right)\hat{j} = (3.25)\cos(61.7)\hat{i} + (3.25)\sin(61.7) = \left[1.54079\hat{i} + 2.86155\hat{j}\right]\frac{m}{s}$$

The acceleration of any object in free fall is 9.81 m/s^2 down or: $\vec{a} = \left[0\hat{i} - 9.81\hat{j}\right]\frac{m}{s^2}$

The uniformly accelerated motion equation we are going to use is: $\vec{r}_f = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

Substituting our knowns into that equation gives us:

$$\Rightarrow \left[x\hat{i} + 0.087\hat{j}\right] = \left[0\hat{i} + 0\hat{j}\right] + \left[1.54079\hat{i} + 2.86155\hat{j}\right]t + \frac{1}{2}\left[-9.81\hat{j}\right]t^2$$

$$\Rightarrow x\hat{i} + 0.087\hat{j} = 1.54079t\hat{i} + 2.86155t\hat{j} - 4.905t^2\hat{j}$$

By isolating the x-direction variables, we cannot solve for anything because we have 2 unknowns:

$$\left(\hat{i}\right) \Rightarrow x = 1.54079t$$

By isolating the y-direction variables, we can solve for time using the quadratic formula:

$$\left(\hat{j}\right) \Rightarrow 0.087 = 2.86155t - 4.905t^2 \Rightarrow -4.095t^2 + 2.86155t - 0.087 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2.86155 \pm \sqrt{(2.86155)^2 - (4)(-4.905)(-0.087)}}{(2)(-4.095)} = \frac{-2.86155 \pm 2.54588}{-9.81}$$

$$\Rightarrow t = 0.291697 \pm 0.259519 = 0.551216\,\text{sec}\ \textit{or}\ 0.0321784\,\text{sec} \Rightarrow t = 0.551216\,\text{sec}$$

We can now return back to the x-direction to solve for the final x-position:

$$\left(\hat{i}\right) \Rightarrow x = 1.54079t = \left(1.54079\right)\left(0.551216\right) = 0.849308m \approx 85cm$$

The basket needs to be placed 85 cm horizontally from the initial x-position of the ball.

But notice all the useful information we can glean from using the quadratic formula:

Remember the quadratic equation $-4.095t^2 + 2.86155t - 0.087 = 0$ is an equation for a parabola. That parabola describes the position of the ball as a function of time. And the quadratic formula gave us this equation for time: $\Rightarrow t = 0.291697\sec \pm 0.259519\sec = 0.551216\sec \; or \; 0.0321784\sec$

- The time to the top of the parabola is 0.291697 or 0.29 sec. This is the time at which the ball reaches its maximum height.
- 0.259519 or 0.26 seconds is how long after the ball reaches its maximum height that the ball reaches a height of 0.087 m.
- 0.259519 or 0.26 seconds is how long *before* the maximum height the ball reaches a height of 0.087 m.
- In other words, the ball is at a height of 0.087 m *twice* after launch, once 0.0321784 or 0.032 seconds after launch while the ball is going up, and once 0.551216 or 0.55 seconds after launch while the ball is going down.
- In order for the ball to "land in the basket" the ball must be moving down, therefore, it must be the larger of the time times, the second time the ball is at a height of 0.087 meters and the ball is moving downward, that it reaches the basket. That is how we know the time to use is 0.551216 seconds.