Flipping Physics Lecture Notes:

Work Energy Theorem Example Problem
http://www.flippingphysics.com/work-energy-theorem-example.html

A 5.0 kg mass experiences a net force causing the object to go from an initial velocity of $\left[4.0\hat{i}-3.0\hat{j}\right]\frac{m}{s}$ to a final velocity of $\left[2.0\hat{i}+6.0\hat{j}\right]\frac{m}{s}$. What is the net work acting on the mass during this event?

We have two equations for work: $W=\vec{F}\bullet\Delta\vec{r}\ \&\ W=\int_{x_i}^{x_f}F_x\,dx$ The dot product equation we use for constant forces. The integral equation we use for non-constant forces. But neither of these equations has mass or velocity in them… So, uh what?

We need to remember that we have what is typically called the Work Energy Theorem. I prefer to call it the Net Work equals Change in Kinetic Energy Theorem: $W_{net}=\Delta KE$

I prefer to call it the Net Work equals Change in Kinetic Energy Theorem because too many of my students forget it is the **net** work and the **change** in **kinetic** energy.

Now let's solve the problem:

$$m=5.0kg;\ v_i=\left[4.0\hat{i}-3.0\hat{j}\right]\frac{m}{s};\ v_f=\left[2.0\hat{i}+6.0\hat{j}\right]\frac{m}{s};\ W_{net}=?$$

$$W_{net}=\Delta KE=KE_f-KE_i=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2=\frac{1}{2}m\left(v_f^2-v_i^2\right)$$

$$\Rightarrow W_{net}=\frac{1}{2}(5)\left[\left(2\hat{i}+6\hat{j}\right)^2-\left(4\hat{i}-3\hat{j}\right)^2\right]$$

But how do we square unit vectors? Well, we use the FOIL method. It looks like this:

$$\left(x\hat{i}+y\hat{j}\right)^2=\left(x\hat{i}+y\hat{j}\right)\bullet\left(x\hat{i}+y\hat{j}\right)=\left(x\hat{i}\right)\bullet\left(x\hat{i}\right)+\left(x\hat{i}\right)\bullet\left(y\hat{j}\right)+\left(y\hat{j}\right)\bullet\left(x\hat{i}\right)+\left(y\hat{j}\right)\bullet\left(y\hat{j}\right)=x^2+0+0+y^2=x^2+y^2$$

This works out to be that we just square each of the unit vector coefficients, however, it is important you realize why that is.

$$\Rightarrow W_{net}=\frac{1}{2}(5)\left[\left(2^2+6^2\right)-\left(4^2-3^2\right)\right]=\frac{1}{2}(5)\left[40-7\right]=82.5\approx 82J$$

(incorrect!)

This includes a mistake my students typically make. Realize, the negative in the initial velocity in the y-direction needs to be included in the square!

$$\Rightarrow W_{net}=\frac{1}{2}(5)\left[\left(2\hat{i}+6\hat{j}\right)^2-\left(4\hat{i}+\left(-3\hat{j}\right)\right)^2\right]$$

$$\Rightarrow W_{net}=\frac{1}{2}(5)\left[\left(2^2+6^2\right)-\left(4^2+\left(-3\right)^2\right)\right]=\frac{1}{2}(5)\left[40-25\right]=37.5\approx 38J$$

So, please, do not forget the Net Work equals Change in Kinetic Energy Theorem. It may come in very handy. Oh and, this equation $W_{net}=\Delta KE$ is not on the AP Physics C equation sheet. So, you need to memorize it. No, I do not know why it is not there. So, stop asking me, please.