Flipping Physics Lecture Notes:

Power and Calculus
http://www.flippingphysics.com/power-calculus.html

We have already defined power as: $P_{average} = \frac{W}{\Delta t}$
It is important we remember this is *average* power, or the power delivered over a time period.

Recall the difference between average and instantaneous velocity:

$$\vec{v}_{average} = \frac{\Delta \vec{r}}{\Delta t} \Rightarrow \vec{v}_{instantaneous} = \frac{d\vec{r}}{dt}$$

In order to determine the instantaneous power, or the power delivered at a specific time, we need to use the derivative: $P_{instantaneous} = \frac{dW}{dt}$

And remember the dot product equation for work: $W = \vec{F} \bullet \Delta \vec{r}$

Therefore, for an infinitesimally small amount of work dW, we have: $W = \vec{F} \bullet \Delta \vec{r} \Rightarrow dW = \vec{F} \bullet d\vec{r}$
Therefore we can substitute into the equation for instantaneous power:

$$P_{instantaneous} = \frac{dW}{dt} = \vec{F} \bullet \frac{d\vec{r}}{dt} = \vec{F} \bullet \vec{v} = Fv\cos\theta$$

And we get that instantaneous power is the dot product of force and velocity.
In other words, the power as a function of time is:

$$P(t) = \frac{dW}{dt} = \vec{F} \bullet \vec{v} = Fv\cos\theta$$

But realize, power does not have to be defined as just via work.
Power can be defined as any type of energy transfer:

$$P = \frac{dE}{dt}$$

In other words, power is the rate at which energy enters or leaves a system by any energy transfer mechanism. Like waves, work, heat, electricity, and radiation as we have talked about before[1]. And its units are watts or joules per second because power is the rate at which energy is transferred into or out of a system.

And remember any derivative can be rearranged to form an integral and work causes a change in energy:

$$P = \frac{dE}{dt} \Rightarrow dE = P\,dt \Rightarrow \int_{E_i}^{E_f} dE = \int_{t_i}^{t_f} P\,dt \Rightarrow E\Big]_{E_i}^{E_f} = E_f - E_i = \Delta E = \int_{t_i}^{t_f} P\,dt \Rightarrow W = \int_{t_i}^{t_f} P\,dt$$

So, notice we have two different equations for work.

One which is work over a change in position: $W = \int_{x_i}^{x_f} F_x\,dx$

And another which is work over a change in time: $W = \int_{t_i}^{t_f} P\,dt$

[1] Energy Transferred Into and Out of a System: http://www.flippingphysics.com/energy-transfer-system.html