Flipping Physics Lecture Notes:

Power using Derivative and Unit Vectors - Example
http://www.flippingphysics.com/power-derivative.html

A 0.280 kg object has a position described by the function $\vec{x}(t)=\left[5.00t^3-8.00t^2-30.0t\right]m$.
What is the net power being delivered to the object at 2.00 seconds?

$m=0.280kg;\ \vec{x}(t)=\left[5.00t^3-8.00t^2-30.0t\right]m;\ P_{net}(2.00)=?$

Because this is the power at a specific point in time (2.00 seconds), we are looking for an instantaneous power. Therefore, we are going to use the equation: $P_{net}=\vec{F}_{net}\bullet\vec{v}$

Because we are looking for the net power, we use the net force to find that power.

And we remember Newton's Second Law: $\sum\vec{F}=m\vec{a}$

Therefore, we need equations for velocity and net force as functions of time in terms of unit vectors.

$$\vec{v}(t)=\frac{dx}{dt}=\frac{d}{dt}\left(5.00t^3-8.00t^2-30.0t\right)=\left[15t^2-16t-30\right]\frac{m}{s}\Rightarrow\vec{v}(t)=\left[\left(15t^2-16t-30\right)\hat{i}\right]\frac{m}{s}$$

$$\vec{a}(t)=\frac{dv}{dt}=\frac{d}{dt}\left(15t^2-16t-30\right)=\left[30t-16\right]\frac{m}{s^2}\Rightarrow\vec{a}(t)=\left[\left(30t-16\right)\hat{i}\right]\frac{m}{s^2}$$

Side note: This object is ***not*** experiencing uniformly accelerated motion.
(The acceleration changes as a function of time and therefore is ***not*** uniform.)
Side^2 note: We actually know this from the position equation because its largest exponent is more than 2.

$$\vec{F}_{net}(t)=m\vec{a}=(0.28)(30t-16)\hat{i}=\left[(8.4t-4.48)\hat{i}\right]N$$

And now we can solve for the net power as a function of time:

$$P_{net}(t)=\vec{F}_{net}\bullet\vec{v}=\left[(8.4t-4.48)\hat{i}\right]\bullet\left[\left(15t^2-16t-30\right)\hat{i}\right]=(8.4t-4.48)\left(15t^2-16t-30\right)watts$$

And now we can substitute in 2 seconds to solve for the instantaneous power delivered to the object at 2 seconds:

$$P_{net}(2)=\left[(8.4)(2)-4.48\right]\left[(15)\left(2^2\right)-(16)(2)-30\right]=(12.32)(-2)=-24.64\approx-24.6watts$$

Notice we could do this without unit vectors:

$$P_{net}=\vec{F}_{net}\bullet\vec{v}=F_{net}v\cos\theta=(ma)v\cos\theta$$

$$\Rightarrow P_{net}(t)=(0.28)(30t-16)\left(15t^2-16t-30\right)\cos\theta$$

$$\Rightarrow P_{net}(2)=(0.28)\left[30(2)-16\right]\left[15\left(2^2\right)-16(2)-30\right]\cos\theta$$

$$\Rightarrow P_{net}(2)=\left|\left|-24.64\right|\right|\cos\theta\Rightarrow 24.64\cos\theta$$

This does mean we need to remember that we use the magnitudes of the force and velocity in the equation $P=Fv\cos\theta$, and it is the cosine of theta that determines if the power is positive or negative. That means we need to determine that angle, θ.
In our net power equation, theta is the angle between the direction of the net force acting on the object and the direction of the velocity of the object. From Newton's Second Law we know that the direction of

the net force acting on the object and the direction of the acceleration of the object are the same. So, we need to determine the directions of the velocity and acceleration of the object at 2 seconds.

$$\vec{v}(t)=\left[15t^2-16t-30\right]\frac{m}{s}\Rightarrow \mathrm{v}(2)=(15)(2^2)-16(2)-30=-2\frac{m}{s}$$

$$\vec{a}(t)=\left[30t-16\right]\frac{m}{s^2}\Rightarrow a(2)=(30)(2)-16=44\frac{m}{s^2}$$

The velocity of the object is in the negative x-direction.
The acceleration of the object is in the positive x-direction.
Therefore, the angle between those two directions is 180°.

$$\Rightarrow P_{net}(2)=\left\|-24.64\right\|\cos\theta=24.64\cos(180)=-24.64\approx-24.6\,watts$$

But what does "The net power delivered to the object at 2 seconds equals negative 24.6 watts" mean?

Watts are joules per second, which means the object is having 24.6 joules of energy removed from it every second. While we do not know anything about individual forces, we do know that the net force is opposite in direction from the velocity of the object. Therefore, this object is slowing down and kinetic energy is being removed from the block system.

We know $W_{net}=\Delta KE \;\&\; P_{net}=\frac{dW_{net}}{dt}$, therefore, we know that, at 2 seconds, the kinetic energy of the object is decreasing at a rate of 24.6 joules every second.