



Flipping Physics Lecture Notes:

From Power to Work using an Integral – Example
<http://www.flippingphysics.com/power-integral.html>

The net power delivered to an object is described by the equation: $P_{net} = [4.00t^2 + t] \text{ watts}$
Determine the net work done on the object from 0 to 4.00 seconds.

Knowns: $P_{net} = [4.00t^2 + t] \text{ watts}; \sum W_{0 \rightarrow 4\text{sec}} = ?$

$$W = \int_{x_i}^{x_f} F_x dx \quad (\text{general equation})$$

The general equation for work we have is:

$$W = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos \theta \quad (\text{constant force})$$

The equation for work done by a constant force:

Those is in terms of position not time. We need an equation for work in terms of time.

Remember, power can be defined using energy transfer, any derivative can be rearranged to form an integral¹, and work causes a change in energy.

$$P = \frac{dE}{dt} \Rightarrow dE = P dt \Rightarrow \int_{E_i}^{E_f} dE = \int_{t_i}^{t_f} P dt \Rightarrow E \Big|_{E_i}^{E_f} = E_f - E_i = \Delta E = \int_{t_i}^{t_f} P dt \Rightarrow W = \int_{t_i}^{t_f} P dt$$

Now we can use this equation to solve for the net work done on the object from 0 to 4.00 seconds.

$$\begin{aligned} \sum W_{0 \rightarrow 4\text{sec}} &= \int_0^4 (4t^2 + t) dt = \left[\frac{4t^3}{3} + \frac{t^2}{2} \right]_0^4 = \left[\left(\frac{4t_f^3}{3} + \frac{t_f^2}{2} \right) - \left(\frac{4t_i^3}{3} + \frac{t_i^2}{2} \right) \right]_0^4 = \left(\frac{(4)(4)^3}{3} + \frac{(4)^2}{2} \right) - 0 \\ &\Rightarrow \sum W_{0 \rightarrow 4\text{sec}} = \frac{280}{3} = 93.\bar{3} \approx 93.3 J \end{aligned}$$

And, because net work equals change in kinetic energy, we know the change in kinetic energy from 0 to 4 seconds of the object equals positive 93.3 joules. In other words, because the net work done on the object is positive, the net work done on the object is causing a positive change in the kinetic energy of the object; it is increasing the speed of the object.

¹ An integral is also called an antiderivative.