Flipping Physics Lecture Notes:

Conservation of Momentum Derivation
and Rocket Demonstration
http://www.flippingphysics.com/conservation-of-momentum-derivation.html

The equation for momentum is: $\vec{p} = m\vec{v}$

The units for momentum are: $\frac{kg \bullet m}{s}$

Newton's Second Law in terms of acceleration is: $\sum \vec{F} = m\vec{a}$

Newton's Second Law in terms of momentum[1] is: $\sum \vec{F} = \frac{d\vec{p}}{dt}$

Substituting in mass times velocity for momentum, using the product rule[2], and remembering that acceleration equals the derivative of velocity with respect to time gives us:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt} = \frac{dm}{dt}\vec{v} + m\vec{a}$$

Which means the version of Newton's Second Law in terms of acceleration assumes the mass of the object on which the net force is acting remains constant.

$$\text{when } \frac{dm}{dt} = 0 \Rightarrow \sum \vec{F} = \frac{dm}{dt}\vec{v} + m\vec{a} \Rightarrow \sum \vec{F} = m\vec{a}$$

Defining Newton's Second Law in terms of momentum means the mass of the object on which the net force is acting can change. (Think rockets!)

When the net force acting on an object or a system equals zero, the derivative of momentum with respect to time equals zero, therefore the momentum of the system is not changing, therefore the momentum of the system is conserved. This is conservation of momentum:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = 0 \Rightarrow \sum \vec{p}_i = \sum \vec{p}_f$$

In other words, linear momentum of a system is conserved when all the forces are internal to the system and the net force acting on the system equals zero.

To understand the basics of how this water rocket accelerates upward, please recall my demonstration of throwing a medicine ball while standing on a skateboard and my demonstration of throwing the same medicine ball while sitting in a canoe. In those demonstrations the medicine ball moves to the left with negative momentum, and the skateboard, canoe, and I move to the right with positive momentum. This is

[1] This equation is closer to Newton's original Second Law than the equation which uses acceleration.

[2] The product rule: $\frac{d}{dt}\left(xy\right) = \frac{dx}{dt}y + x\frac{dy}{dt}$

because momentum is conserved when I throw the ball, because the force I apply on the ball is equal and opposite to the force the ball applies on me. In other words, all the forces are internal and the net force on the medicine ball and me equals zero. The same thing is happening in the water rocket. Air pressure inside the rocket is pushing individual molecules of water out the bottom of the rocket, all forces are internal, momentum is conserved, the water molecules have downward momentum and the rocket has upward momentum. Each water molecule is just like a microscopic medicine ball being pushed downward by the rocket causing the rocket to move upward. And, as the water leaves the rocket, the mass of the rocket decreases.

In reality the net force acting on the rocket and water system does not quite equal zero because there is a force of gravity acting downward on it, so the net force acting on the system equals the downward force of gravity acting on the rocket. However, we are considering the force of gravity to be small enough relative to the internal forces acting within the system such that the force of gravity is negligible. And you can see that is true because the rocket moves upward quite quickly.