



## Flipping Physics Lecture Notes:

The Scalar Nature of Variables in Rotational Motion Equations  
<http://www.flippingphysics.com/scalar-rotational-variables.html>

We have already discussed the following equations:

$$s = r\Delta\theta; v_t = r\omega; a_t = r\alpha; a_c = \frac{v_t^2}{r} = r\omega^2$$

It is important that the vector symbol is never used in any of those equations. Let's look at one of them specifically and identify what this means.

$$v_t = r\omega$$

Typically, I have said this equation as tangential velocity equals radius times angular velocity. Let's look at what it would mean if this equation did have vector symbols on it.

$$\vec{v}_t = r\vec{\omega}$$

Because radius,  $r$ , is a scalar, this equation would imply that the tangential velocity and angular velocity are in the same direction. Hopefully you recognize that the tangential velocity and angular velocity of an object cannot be in the same direction. The tangential velocity is directed tangent to the circle the object is moving along, and the object's angular velocity is normal to the two-dimensional plane the object is moving along and those cannot be the same direction. Therefore, this equation cannot refer to the vector quantities and must refer to the magnitudes of the vector quantities.

In other words, this equation,  $v_t = r\omega$ , needs to be read as the magnitude of tangential velocity equals radius times the magnitude of angular velocity. Alternatively, it could also be read as tangential speed equals radius times angular speed.

So, realize all of these equations refer to the magnitudes of the vectors. For example:

- $s = r\Delta\theta \rightarrow$  Arc length equals radius times the magnitude of angular displacement.
- $v_t = r\omega \rightarrow$  Tangential speed equals radius times angular speed.
- $a_t = r\alpha \rightarrow$  The magnitude of tangential acceleration equals radius times the magnitude of angular acceleration.

- $a_c = \frac{v_t^2}{r} = r\omega^2 \rightarrow$  The magnitude of centripetal acceleration equals tangential speed

squared divided by radius and it also equals radius times angular speed squared.

I know I have not been overly clear about this before, and I apologize for that. Hopefully this clears this up a bit. ☺