Flipping Physics Lecture Notes:

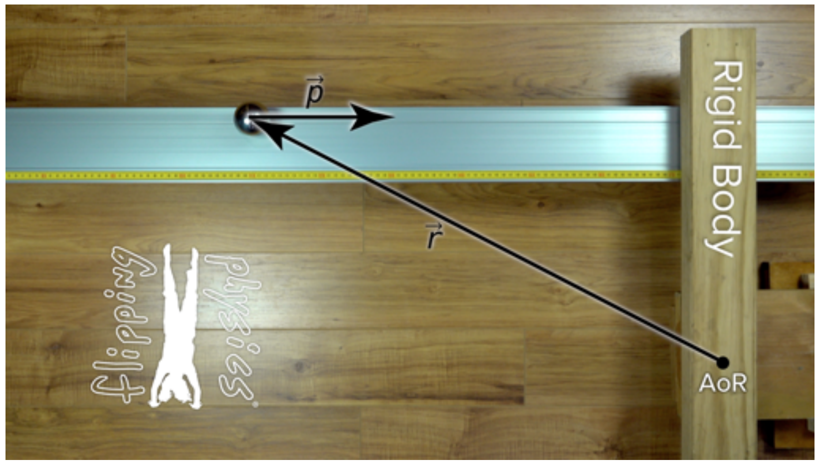
Cross Product Angular Momentum Derivation
http://www.flippingphysics.com/angular-momentum-cross-product.html

Let's start with Newton's Second Law:

$$\sum \vec{F} = m\vec{a} \Rightarrow \sum \vec{\tau} = I\vec{\alpha}$$

But we also have a version of Newton's Second Law using linear momentum:

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$



Now, let's say we have a ball moving toward a rigid body with an axis of rotation. The ball has **r**, a position vector relative to the axis of rotation of the rigid body, and the ball has linear momentum, **p**.

Let's take the cross product of Newton's Second Law with the position vector:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \vec{r} \times \left(\sum \vec{F} = \frac{d\vec{p}}{dt}\right) \Rightarrow \vec{r} \times \sum \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}$$

The left side of the equation is net torque: $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \sum \vec{\tau} = \vec{r} \times \sum \vec{F}$

And $\frac{d\vec{r}}{dt} = \vec{v}$ and $\vec{v}$ & $\vec{p}$ are $\parallel \Rightarrow \frac{d\vec{r}}{dt} \times \vec{p} = 0$

Therefore, our original cross product equation becomes:

$$\Rightarrow \sum \vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p} = \frac{d\left(\vec{r} \times \vec{p}\right)}{dt} \quad \text{(Product Rule)}$$

And we can define angular momentum, **L**, as the cross product of the position vector and momentum:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \sum \vec{\tau} = \frac{d\vec{L}}{dt} \quad \text{where } \vec{L} \equiv \vec{r} \times \vec{p}$$

And because momentum equals mass times velocity: $\vec{L} = \vec{r} \times m\vec{v} \Rightarrow L = rmv\sin\theta$[♥]

Lastly, realize the rigid body does not need to be there for a moving object to have angular momentum about a point.

♥ "Angular Momentum of Particles Introduction" https://www.flippingphysics.com/angular-momentum-particles.html