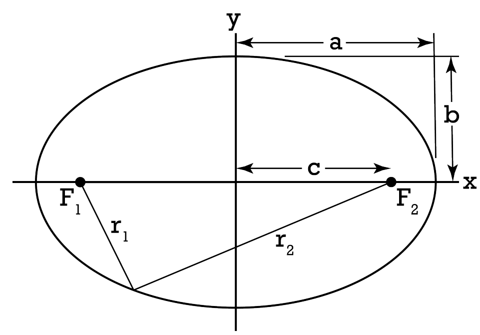
Flipping Physics Lecture Notes:

Kepler's First Law of Planetary Motion

http://www.flippingphysics.com/kepler-first.html

For thousands of years the geocentric model of the solar system (Earth at the center with the planets and Sun revolving in circles around the Earth) was considered to be correct. This model was formalized by the Greek astronomer Claudius Ptolemy (c. 100 – c. 170 CE). In 1543, Roughly 1400 years later, Nicolaus Copernicus (1473 – 1543), a Polish astronomer postulated a heliocentric model (Sun at the center with planets revolving in circular orbits). The Danish astronomer Tycho Brahe (1546 – 1601) made accurate and comprehensive observations of the locations of planets and stars in the sky for 20 years before his death in 1601. These observations were done without a telescope because it was not invented until the early 1600s. After Tycho Brahe's death, his assistant, Johannes Kepler (1571 – 1630) a German astronomer, analyzed Tycho Brahe's data for roughly 15 years and derived mathematical descriptions of planetary motion now known as Kepler's Three Laws of Planetary Motion.

Kepler's First Law of Planetary Motion states:
"Planets move in elliptical orbits with the Sun at one focus."



This can be illustrated with the illustration at right. ☺ An ellipse has two foci. I have labelled them as F_1 and F_2. The curve is drawn such that the sum of r_1 and r_2, the distances from the respective foci to the curve, is constant: $r_1 + r_2 = \text{constant}$.

- The **major axis** is the longest distance through the center which also passes through both foci and is the distance 2a.
- The **semimajor axis** is the distance a.
- The **minor axis** is the shortest distance through the center is the distance 2b.
- The **semiminor** axis is the distance b.
- Each focus is located a distance c[1] from the center of the ellipse.
- Now that we have defined the major axis, we know $r_1 + r_2 = 2a$.

The distance from the location where $r_1 = r_2$ to either focal point is equal to a. That means, for an ellipse, $c^2 + b^2 = a^2$. (Not to be confused with the Pythagorean theorem. ☺)

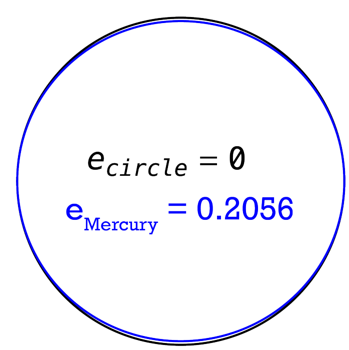
The eccentricity of an orbit is defined as: $e = \frac{c}{a}$

For a circular orbit c = 0, therefore

$$e_{circle} = \frac{0}{a} = 0$$

The eccentricities of the 8 planets are shown in the table:[2] Notice the largest eccentricity here is Mercury at 0.2056. I have illustrated what that looks like. It is nearly a circle, which is why we often approximate planetary orbits as circular.

Planet:	Eccentricity:
Venus	0.0068
Neptune	0.0086
Earth	0.0167
Uranus	0.0472
Jupiter	0.0484
Saturn	0.0542
Mars	0.0934
Mercury	0.2056



To clarify why Mercury's orbit appears to be almost circular:

$$e = 0.2056 = \frac{c}{a} \Rightarrow c = 0.2056a \;\;\&\;\; a^2 = b^2 + c^2$$

$$\Rightarrow b^2 = a^2 - c^2 = a^2 - (0.2056a)^2 = a^2 - 0.042271a^2 = 0.95773a^2$$

$$\Rightarrow b = \sqrt{0.95773a^2} = 0.97864a \approx 0.98a$$

And realize an ellipse has eccentricities between 0 and 1: $0 \le e_{ellipse} < 1$

[1] Not all textbooks agree on the letters used for semimajor axis, semiminor axis, and distance from ellipse center to foci. Sorry!
[2] Data from https://www.astronomynotes.com/tables/tablesb.htm