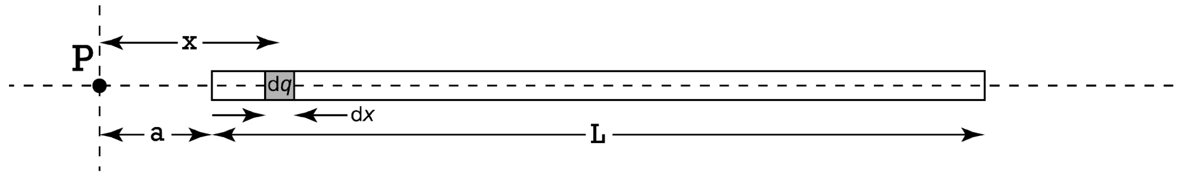
Flipping Physics Lecture Notes:
Charged Rod Electric Field
http://www.flippingphysics.com/rod-electric-field.html

Determine the electric field at point P, which is located a distance “a” to the left of a thin rod with a charge +Q, uniform charge density λ, and length L.



Notice that, if we were to place a positive point charge at point P, it would experience a force to the left from every dq or every infinitesimally small part of the wire. Therefore, we already know the direction of the electric field at point P, it will be to the left or in the negative “i" direction. Now let’s solve for the electric field:

$$\vec{E} = k\int \frac{dq}{r^2}\hat{r} \Rightarrow \vec{E} = k\int \frac{dq}{x^2}\left(-\hat{i}\right) \;\&\; \lambda = \frac{Q}{L} = \frac{dq}{dx} \Rightarrow dq = \lambda dx \;\&\; Q = \lambda L$$

$$\Rightarrow \vec{E} = -k\hat{i}\int \frac{\lambda dx}{x^2} = -k\lambda\hat{i}\int_a^{a+L}\left(\frac{1}{x^2}\right)dx = -k\lambda\hat{i}\int_a^{a+L}\left(x^{-2}\right)dx$$

$$\Rightarrow \vec{E} = -k\lambda\hat{i}\left[\frac{x^{-1}}{-1}\right]_a^{a+L} = k\lambda\hat{i}\left[\frac{1}{x}\right]_a^{a+L} = k\lambda\hat{i}\left[\frac{1}{a+L} - \frac{1}{a}\right] = k\lambda\hat{i}\left[\frac{a-(a+L)}{a(a+L)}\right]$$

$$\Rightarrow \vec{E} = k\lambda\hat{i}\left[\frac{a-a-L}{a(a+L)}\right] = -\frac{k\lambda L}{a(a+L)}\hat{i} \Rightarrow \vec{E} = -\frac{kQ}{a(a+L)}\hat{i}$$

$$\text{if } a >> L \text{ then } a + L \approx a \;\&\; \vec{E} = -\frac{kQ}{a^2}\hat{i}$$

In other words, if we get far enough from the charged rod, it acts like a point charge. ☺