



Flipping Physics Lecture Notes:  
Irregularly Shaped Conductors in Electrostatic Equilibrium  
<http://www.flippingphysics.com/electrostatic-equilibrium-irregular-shape.html>

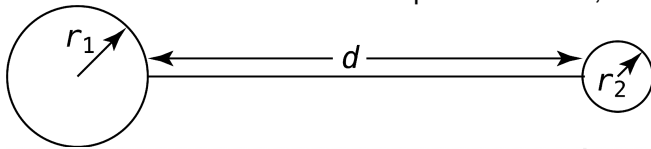
Conductors are materials where the electrons are free to move rather easily, however, when they are in electrostatic<sup>1</sup> equilibrium, this means the charges are stationary in the object. There are four things you need to remember about conductors in electrostatic equilibrium. The first three are described in my video:

- “3 Properties of Conductors in Electrostatic Equilibrium”
- <http://www.flippingphysics.com/electrostatic-equilibrium.html>

The fourth is:

For an irregular shape, the local surface charge density is at its maximum where the radius of curvature is at its minimum. In other words, the largest number of excess charges per area will be where the radius of curvature is the smallest.  $\sigma_{\text{local}} = \text{maximum @ } r_{\text{curvature}} = \text{minimum}$

- a. To prove this, we have two conducting spheres connected by a long conducting wire with the whole system in electrostatic equilibrium.
  - i. This system is a conductor in electrostatic equilibrium. In other words, when two conductors are brought into contact with one another, the charges redistribute such that both conductors are at the same electric potential. Please realize this happens so quickly that the time for this to occur is considered to be negligible.
- b. The radius of sphere 2 is smaller than the radius of sphere 1, and the distance,  $d$ , between the two spheres is much, much larger than either radius.



$$r_2 < r_1 \ \& \ d \gg r_1 \ \& \ V_1 = V_2 \Rightarrow \frac{kq_1}{r_1} = \frac{kq_2}{r_2} \Rightarrow \frac{q_1}{r_1} = \frac{q_2}{r_2}$$

$$\Rightarrow q_1 = \left(\frac{r_1}{r_2}\right) q_2 \ \& \ \frac{r_1}{r_2} > 1 \Rightarrow q_1 > q_2$$

$$E_1 = \frac{kq_1}{(r_1)^2} \ \& \ E_2 = \frac{kq_2}{(r_2)^2} \Rightarrow \frac{E_1}{E_2} = \frac{\frac{kq_1}{(r_1)^2}}{\frac{kq_2}{(r_2)^2}} = \left(\frac{kq_1}{(r_1)^2}\right) \left(\frac{(r_2)^2}{kq_2}\right) = \frac{q_1 (r_2)^2}{q_2 (r_1)^2}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{\left(\left(\frac{r_1}{r_2}\right) q_2\right) (r_2)^2}{q_2 (r_1)^2} = \frac{r_2}{r_1} \Rightarrow E_2 = \left(\frac{r_1}{r_2}\right) E_1 \ \& \ \frac{r_1}{r_2} > 1$$

$$\Rightarrow E_2 > E_1 \ \& \ E = \frac{\sigma_{\text{local}}}{\epsilon_0} \Rightarrow \sigma_2 > \sigma_1 \Rightarrow \text{if } r_2 < r_1 \text{ then } \sigma_2 > \sigma_1$$

<sup>1</sup> Electrostatics is the study of electromagnetic phenomena that occur when there are no moving charges.