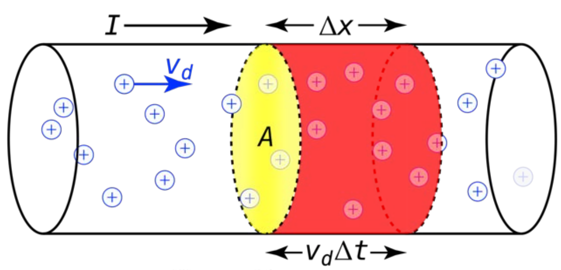
Flipping Physics Lecture Notes:
Current, Resistance, and Simple Circuits
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-current-resistance-simple-circuits.html

Electric current, I, is defined as the derivative of charge with respect to time: $I \equiv \frac{dq}{dt}$

- $I \equiv \frac{dq}{dt} \Rightarrow \frac{\text{coulombs}, C}{\text{seconds}, s} = \text{amperes}, A$
    - Amperes are a base S.I. unit.
    - This is instantaneous current.
- $I_{\text{average}} = \frac{\Delta Q}{\Delta t}$
- Current is the electric charge of the charges which pass by a point in a current carrying wire divided by the time it takes for those charges to pass by that point.
- Current occurs when there is an electric potential difference across a wire. If there is no electric potential difference, current does not flow.
    - $\Delta V = 0 \Rightarrow I = 0$

Unless otherwise stated, electric current in this class is all considered to be *conventional current*:

- The direction of conventional current is the direction positive charges *would* flow.
- Reality is that, in most circuits, negative charge carries (electrons, e^-) move opposite the direction of conventional current.



Let's look at charges flowing in a wire:
Start with the average current over a small section of the wire Δx:

- $I_{\text{average}} = \frac{\Delta Q}{\Delta t}$
- $\Delta Q = (\text{\# of charge carriers})(\text{charge per carrier}, q)$
- Charge carrier density, n:
- $n = \frac{\text{\# of charge carriers}}{\text{volume}, V} \Rightarrow \text{\# of charge carriers} = nV$
  $\Rightarrow \Delta Q = nVq \;\;\&\;\; V = A\Delta x \Rightarrow \Delta Q = nA\Delta x q$
- $v_{\text{drift}} = v_d = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v_d \Delta t$
    - Drift velocity, v_d: The average velocity of the charge carriers in a current carrying wire.
        - If the current is zero, the charge carriers are still moving, however, the average velocity of the charge carriers is zero.
        - v_d typically is quite low. On the order of 0.1 mm/s. The reason lightbulbs in a circuit (for example) turn on immediately when you flip the switch is because all the electrons are already in the wire. When you flip the switch, they all start flowing.
- $\Rightarrow \Delta Q = nAv_d\Delta t q \Rightarrow I = \frac{\Delta Q}{\Delta t} = \frac{nAv_d\Delta t q}{\Delta t} \Rightarrow I = nAv_d q$

Current density, J, is current per unit area:

- $J = \frac{I}{A} = \frac{nAv_d q}{A} \Rightarrow J = nv_d q$ & $J = \sigma E$
    - Materials which have this property are considered to be ohmic and follow Ohm's Law.
    - σ is the conductivity of the material.
        - Conductivity is a measure of how little a material opposes the movement of electric charges.
        - Conductivity is a fundamental property of a material.
- $\|\Delta V\| = Ed \Rightarrow \|\Delta V\| = EL$
    - An electric potential difference across a wire is what causes current in the wire and we are assuming the electric field created in the wire is uniform. Rather than using d for the distance in the electric field, we use L for the length of the wire.
- $\Rightarrow E = \frac{\Delta V}{L} \Rightarrow J = \sigma\left(\frac{\Delta V}{L}\right) \Rightarrow \Delta V = \frac{JL}{\sigma} = \frac{IL}{A\sigma} \Rightarrow \Delta V = \left(\frac{L}{\sigma A}\right)I$

The *resistance* of a wire, R, is defined as $R = \frac{L}{\sigma A}$

- However, usually resistance is defined in terms of *resistivity*, ρ.
    - Resistivity is a measure of how strongly a material opposes the movement of electric charges.
    - Resistivity is a fundamental property of a material.
    - $\rho = \frac{1}{\sigma} \Rightarrow R = \frac{\rho L}{A}$ & $E = \rho J$
    - This equation requires the resistor to have uniform geometry.
- Which brings us to the more common version of Ohm's law:
    - $\Delta V = \left(\frac{L}{\sigma A}\right)I = I\left(\frac{\rho L}{A}\right) \Rightarrow \Delta V = IR$
    - Again, not all materials are ohmic and follow Ohm's law.
- $\Rightarrow R = \frac{\Delta V}{I} \Rightarrow \text{ohms, } \Omega = \frac{\text{volts, } V}{\text{amperes, } A}$
- $R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L} \Rightarrow \frac{\Omega \cdot m^2}{m} = \Omega \cdot m$

Resistance and *resistivity* are two terms which students often mix up:

- Resistance has units of ohms, Ω, and is a property of an object.
- Resistivity has units of Ω·m and is property of a material.
- Two objects can have the same resistivity but different resistances if they are made of the same material; however, they have different lengths or cross-sectional areas.

The resistivity of a conducting material typically decreases with decreasing temperature. Think of superconductors. Superconducting materials have zero resistivity, and require very, very low temperatures.

- In this class, unless otherwise stated, the resistivity of conducting materials is considered to be constant regardless of temperature.
- Resistors usually convert electric potential energy to thermal energy which can increase the temperature of the resistor and can increase the temperature of the resistor's environment.

Now we get to discuss *electric power*, which is the rate at which electric potential energy is converted to other types of energy such as heat, light, and sound.

$$P = \frac{dU}{dt} \Rightarrow P_{elec} = \frac{dU_{elec}}{dt} = \frac{d(q\Delta V)}{dt} = \frac{dq}{dt}\Delta V \Rightarrow P = I\Delta V$$

$$\& \;\; \Delta V = IR \Rightarrow P = I(IR) = I^2R$$

$$\& \;\; I = \frac{\Delta V}{R} \Rightarrow P = \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2}{R}$$

$$\Rightarrow P = I\Delta V = I^2R = \frac{\Delta V^2}{R}$$

A unit which is often used when it comes to electricity is the kilowatt-hour:

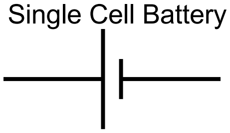
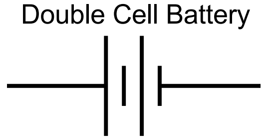
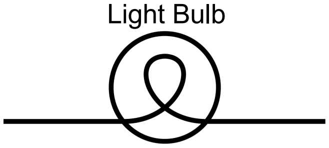
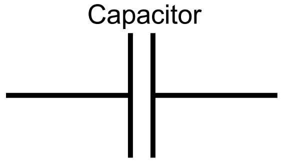
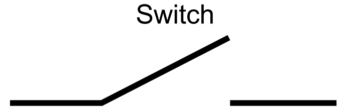
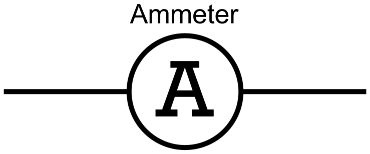
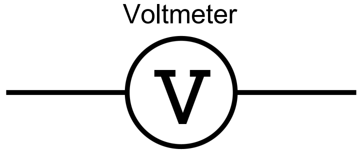
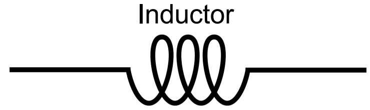
$$1kW{\cdot}hr\left(\frac{1W}{1000kW}\right) = 1000W{\cdot}hr = 1000\left(\frac{J}{s}\right)hr\left(\frac{3600s}{1hr}\right) = 3.6\times10^6 J$$

In other words, the kilowatt-hour is a misnomer (or maybe just misleading). It sounds like a unit of power; however, it is a unit of energy. And we know: $1kW \cdot hr = 3.6MJ$

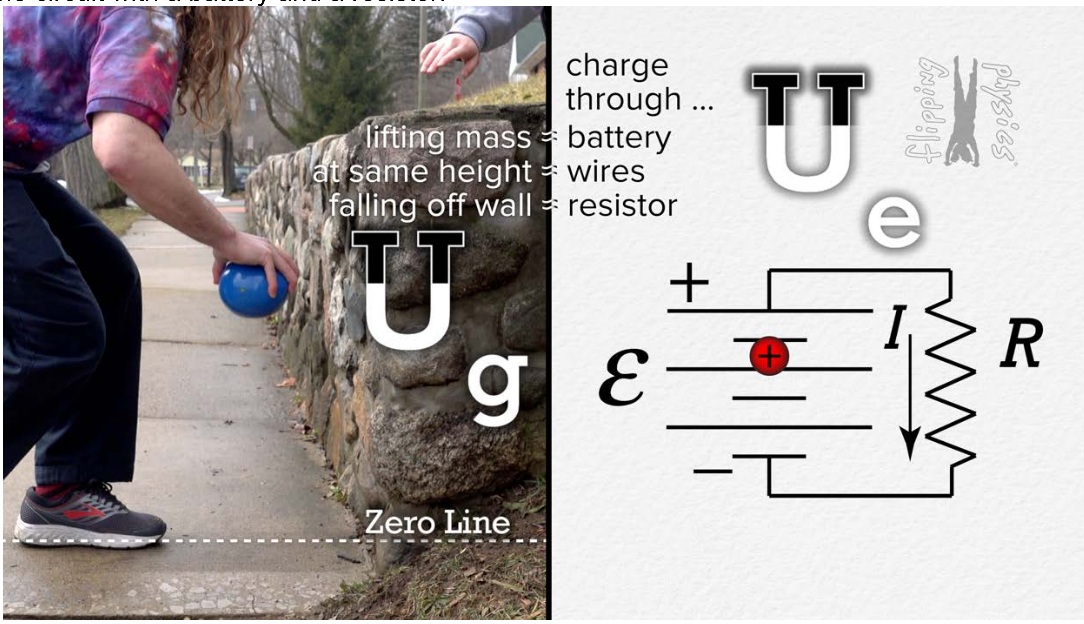
A light bulb is a common item used in physics. It is a resistor which converts electric potential energy to light, heat, and sound energy. The brightness of a light bulb increases with increasing power; therefore, the brightness of a light bulb is often used to demonstrate the power in an electric circuit. Speaking of electric circuits…

The Basics of Electric Circuits:

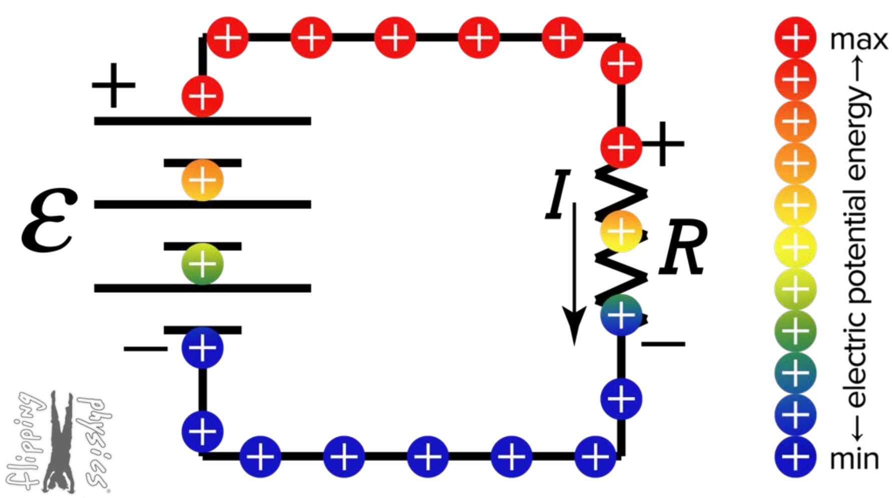
- An electric circuit is typically composed of electrical loops which can include wires, batteries, resistors, light bulbs, capacitors, switches, ammeters, voltmeters, and inductors.
- Typical symbols for elements in electric circuits are:

Single Cell Battery 	Double Cell Battery 	Resistor 
Light Bulb 	Capacitor 	Switch 
Ammeter 	Voltmeter 	Inductor 

A simple circuit with a battery and a resistor:



- The long line of the battery is the positive terminal, and the short line is the negative terminal.
- *Electromotive force*, emf, ε, is the ideal electric potential difference, or voltage, across the terminals of the battery.
    - Yes, the symbol, lowercase Greek letter epsilon, is the same as electric permittivity. ☹
    - Yes, electromotive force is not a force. The term is another misnomer. ☹
- According to the law of charges, positive charges are repelled from the positive terminal and attracted to the negative terminal; therefore, current is clockwise in this circuit.
- A battery adds electric potential energy to electric charges.
    - Like lifting a mass adds gravitational potential energy to masses.
    - A battery is essentially an electric potential energy pump.
- A resistor converts electric potential energy to heat energy. (And maybe light sound energy)
    - Like a mass falling off a wall converts gravitational potential energy to kinetic energy.
- Unless otherwise stated, wires are considered to be ideal and have zero resistance; therefore, there is no change in electric potential energy of charges as they move along a wire.
    - Like a mass at rest maintaining a constant height and therefore a constant gravitational potential energy at either the top or bottom of the wall.



- *Terminal Voltage*, ΔV_t, is the measured voltage across the terminals of the battery.
  - Because all real batteries have some internal resistance, when a battery is supplying current to a circuit, the terminal voltage of a real battery is less than the emf.
    - The symbol for the internal resistance of a real battery is typically, r.
  - One way to illustrate a real battery in an electric circuit is shown in yellow.
  - $\Delta V_t = \varepsilon - \Delta V_r \Rightarrow \Delta V_t = \varepsilon - Ir$
  - As current increases, the terminal voltage decreases.
  - The only way to get the terminal voltage to be equal to the emf is to have no current flowing through the battery.