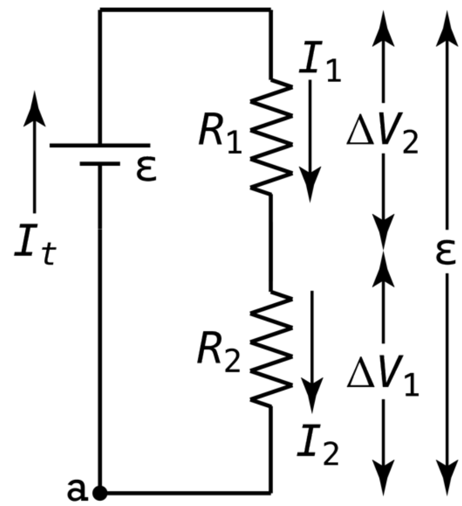
Flipping Physics Lecture Notes:
Series and Parallel Circuits
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-series-parallel-circuits.html

When an anthropomorphic[1] charge has no choice but to go through two circuit elements, those two circuit elements are in *series*. For example, a charge which goes through resistor 1 has no choice but to also go through resistor 2. There is no other path for the anthropomorphic charge to choose.



The currents through the three circuit elements must all be equal:

$$I_t = I_1 = I_2$$

The “t” in the subscript refers to the current at the terminals of the battery which is the current delivered by the battery to the circuit.

The electric potential difference across the battery equals the summation of the electric potential difference across the two resistors:

$$\Delta V_{\text{bottom wire}\rightarrow\text{top wire}} = \varepsilon = \Delta V_1 + \Delta V_2$$

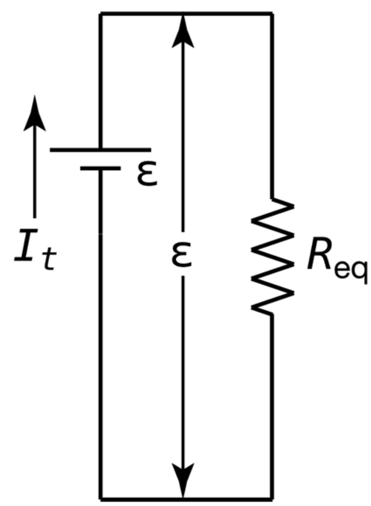
(If you’d prefer to look at this in terms of the electric potential difference around the loop in the circuit:)

$$\Delta V_{\text{loop}} = V_f - V_i = V_a - V_a = 0 = \varepsilon - \Delta V_1 - \Delta V_2 \Rightarrow \varepsilon = \Delta V_1 + \Delta V_2$$

We know Ohm’s law: $\Delta V = IR$; therefore, …

$$\Rightarrow \varepsilon = I_t R_{eq} = I_1 R_1 + I_2 R_2$$

$$\Rightarrow R_{eq} = R_1 + R_2$$

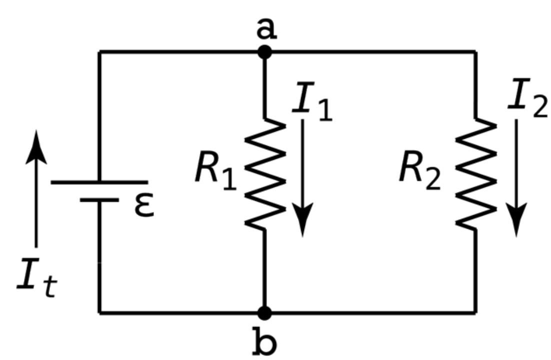


The “eq” in the subscript means equivalent. In other words, R_{eq} is one resistor with the equivalent resistance of the two resistors.

Therefore, the equation for the equivalent resistance of n resistors in series is:

$$R_{\text{eq series}} = \sum_n R_n = R_1 + R_2 + \ldots$$

When an anthropomorphic charge has the choice between two circuit elements and then the paths through those two circuit elements reconverge without going through another circuit element, the two circuit elements are in *parallel*.



When circuit elements are in parallel, their electric potential differences are equal:

$$\varepsilon = \Delta V_1 = \Delta V_2$$

Note the junctions at points a and b. Due to conservation of charge, the net current going into a junction equals the net current coming out of a junction. For junction a:

[1] *Anthropomorphism:* Giving human characteristics or behaviors to non-human objects.

$$I_{\text{in}} = I_{\text{out}} \Rightarrow I_t = I_1 + I_2$$

We can then use Ohm's law:

$$\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow \frac{\varepsilon}{R_{\text{eq}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

And we get the equivalent resistance for the two resistors in parallel:

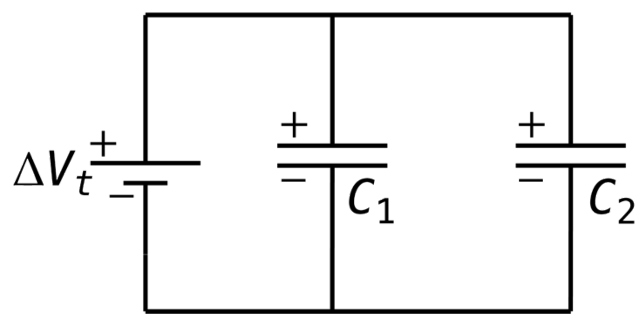
$$\Rightarrow R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

And the equivalent resistance for n resistors in parallel:

$$\Rightarrow R_{\text{eq parallel}} = \left(\sum_n \frac{1}{R_n}\right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}$$

When we add a resistor in series, the equivalent resistance increases.
When we add a resistor in parallel, the equivalent resistance decreases.

Now let's look at two capacitors in parallel:
We know the electric potential differences are all equal.



$$\Delta V_t = \Delta V_1 = \Delta V_2$$

Because the charges moved to the top plates of the capacitors need to go to either capacitor 1 or capacitor 2, the charge moved by the battery to the plates of the capacitors equals the sum of the charges on the capacitors:

$$Q_t = Q_1 + Q_2$$

We can then use the definition of capacitance:

$$C = \frac{Q}{\Delta V} \Rightarrow Q = C\Delta V$$

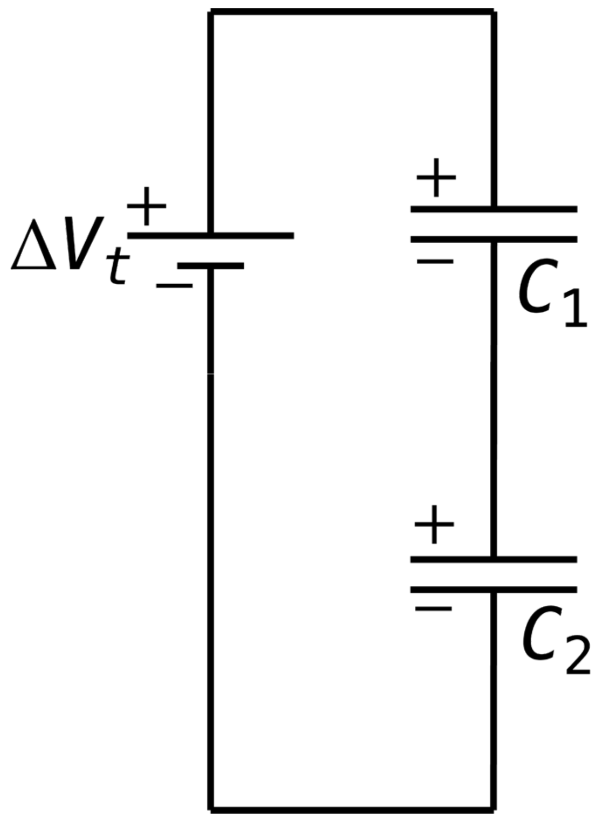
To derive the equivalent capacitance of two capacitors in parallel:

$$\Rightarrow C_{\text{eq}}\Delta V_t = C_1\Delta V_1 + C_2\Delta V_2 \Rightarrow C_{\text{eq}} = C_1 + C_2$$

And the equivalent capacitance of n capacitors in parallel:

$$\Rightarrow C_{\text{eq parallel}} = \sum_n C_n = C_1 + C_2 + \ldots$$

And we can now look at two capacitors in series:



The electric potential is the same as resistors in series:

$$\Delta V_t = \Delta V_1 + \Delta V_2$$

And the charges on each capacitor are equal:

$$Q_t = Q_1 = Q_2$$

This is because the magnitude of the charge moved by the battery to the top plate of capacitor 1 and the bottom plate of capacitor 2 are equal in magnitude. And those plates polarize the charges on the wire between the two capacitors and the bottom of capacitor 1 and the top of capacitor 2. This causes all four plates of the two capacitors to have equal magnitude charges. This is an illustration of conservation of charge.

And we can solve for electric potential difference in terms of capacitance and charge:

$$Q = C\Delta V \Rightarrow \Delta V = \frac{Q}{C}$$

And use that to solve for the equivalent capacitance of two capacitors:

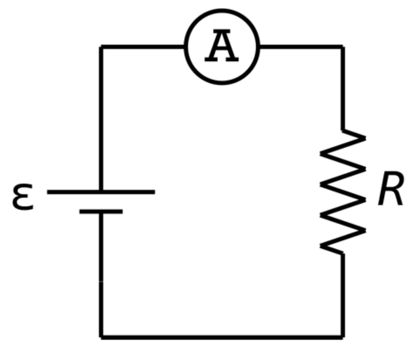
$$\Rightarrow \frac{Q_t}{C_{\text{eq}}} = \frac{Q_1}{C_2} + \frac{Q_2}{C_2} \Rightarrow \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

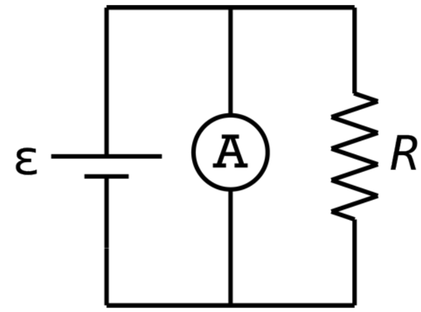
And the equivalent capacitance of n capacitors:

$$\Rightarrow C_{\text{eq series}} = \left(\sum_n \frac{1}{C_n}\right)^{-1} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right)^{-1}$$

Notice the equations for resistors and capacitors are reversed. That means that:
When we add a capacitor in parallel, the equivalent capacitance increases.
When we add a capacitor in series, the equivalent capacitance decreases.

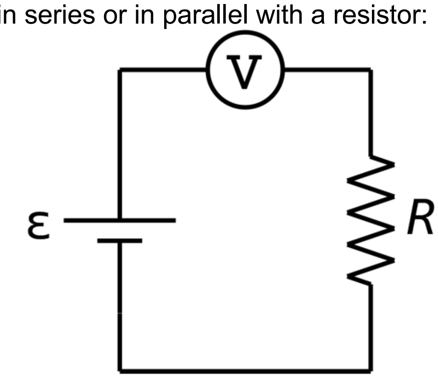
Let’s discuss how to use the tools which measure current and electric potential difference. Starting with the ammeter which measures current or amperes. We need to decide if an ammeter needs to be put in series or parallel with the circuit element it is meant to measure the current through. So, let’s look at what happens when we attempt to measure the current through a resistor using an ammeter in series and in parallel with a resistor:

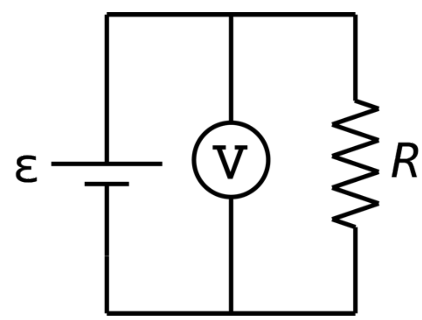




Hopefully you recognize that placing an ammeter in parallel with a resistor will not measure the current through the resistor because the current through the ammeter and the resistor are not the same. Therefore, an ammeter needs to be placed in series with a circuit element to measure the current through that circuit element. Also, the resistance of an ammeter needs to be *very* small. In the above example, if the resistance of the ammeter is not *very* small, it will increase the equivalent resistance of the circuit and decrease the current through the resistor you are trying to measure the current through. Unless otherwise indicated, ammeters in this class are considered to have zero resistance.

And now let's attempt to measure the electric potential difference across a resistor using a voltmeter either in series or in parallel with a resistor:





Hopefully you recognize that placing a voltmeter in series with a resistor will not measure the electric potential difference across the resistor because the voltage across the voltmeter and the resistor are not the same. Therefore, a voltmeter needs to be placed in parallel with a circuit element to measure the voltage across that circuit element. Also, the resistance of a voltmeter needs to be *very* large. In the above example, if the resistance of the voltmeter is not *very* large, it will decrease the equivalent resistance of the circuit, increase the current delivered by the battery, and change the overall properties of the circuit. Unless otherwise indicated, voltmeters in this class are considered to have infinite resistance.

To review:

● Ammeters:	● Voltmeters:
○ Measure current	○ Measure electric potential difference
○ Placed in *series* with the circuit element	○ Placed in *parallel* with circuit element
○ Have nearly *zero* resistance*	○ Have nearly *infinite* resistance*

* You may see this called impedance in product literature for Voltmeters and Ammeters, due to the fact that there is more to the behavior of these devices than just resistance. For the purpose of this class and the AP Physics C Electricity and Magnetism exam, it will be called resistance unless otherwise noted.