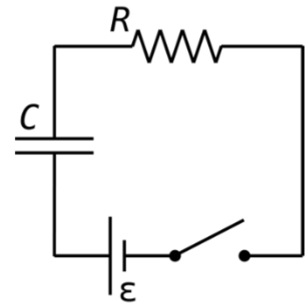
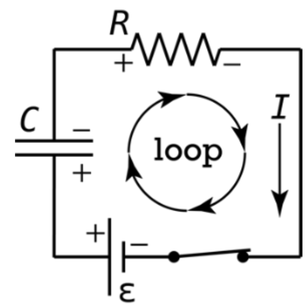
Flipping Physics Lecture Notes:
RC Circuit Basics
http://www.flippingphysics.com/rc-circuit.html



Up until this point we have assumed all changes in electric current, electric potential difference, and charge on capacitor plates were instantaneous. Today, we put a resistor and a capacitor together and learn how those variables change as a function of time. This is called an RC circuit. We start with a circuit composed of an uncharged capacitor, a resistor, a battery, and an open switch, all connected in series.

At time initial, $t_i = 0$, we close the switch.
We are *charging a capacitor through a resistor.*

Let's start by adding a loop in the direction of current flow in the circuit. Then use Kirchhoff's Loop Rule starting in the lower right-hand corner of the circuit:



$$\Delta V_{\text{loop}} = 0 = +\varepsilon - \Delta V_C - \Delta V_R$$

We can use the definition of capacitance to solve for the electric potential difference across the capacitor:

$$C = \frac{Q}{\Delta V} \Rightarrow \Delta V_C = \frac{Q}{C}$$

And we know Ohm's law: $\Delta V_R = IR$

$$\Rightarrow \Delta V_{\text{loop}} = 0 = \varepsilon - \frac{q}{C} - iR$$

Notice we are using lowercase "q" for charge because the charge is changing as a function of time. I wish we had a similar notation for current, I, however, if we used lowercase "i", I am sure it would be more confusing. So, please realize charge, q, and current, I, are both changing as a function of time in the above equation.

Now let's look at limits, starting with $t_i = 0$:

- The initial charge on the capacitor is zero: $q_i = 0$
- This means the initial electric potential difference across the capacitor is also zero:
  $$\Rightarrow \Delta V_{C_i} = \frac{q}{C} = \frac{0}{C} = 0$$
- We can now use the loop equation to solve for the initial current through the circuit.
  $$\Rightarrow 0 = \varepsilon - \frac{0}{C} - i_{\text{initial}}R \Rightarrow i_{\text{initial}}R = \varepsilon \Rightarrow i_{\text{initial}} = \frac{\varepsilon}{R} = i_{\text{max}}$$
- Because the charge on the capacitor will increase as a function of time, electric potential difference across the capacitor will also increase. This means the current in the circuit will decrease. In other words, the initial current in the circuit is also the maximum current.

And now the limit of "after a long time" or the $t_f \approx \infty$.

- The final current in the circuit is zero: $i_{\text{final}} \approx 0$

- This means the final electric potential difference across the resistor is also zero:
  $\Rightarrow \Delta V_{R_f} = i_{\text{final}} R = (0) R = 0$
- And we can use the loop equation to solve for the final charge on the capacitor:
  $\Rightarrow 0 = \varepsilon - \frac{q_f}{C} - (0) R \Rightarrow q_f = \varepsilon C = q_{\text{max}}$
- Because we know the charge has been increasing this whole time, we know this is the maximum charge on the capacitor.