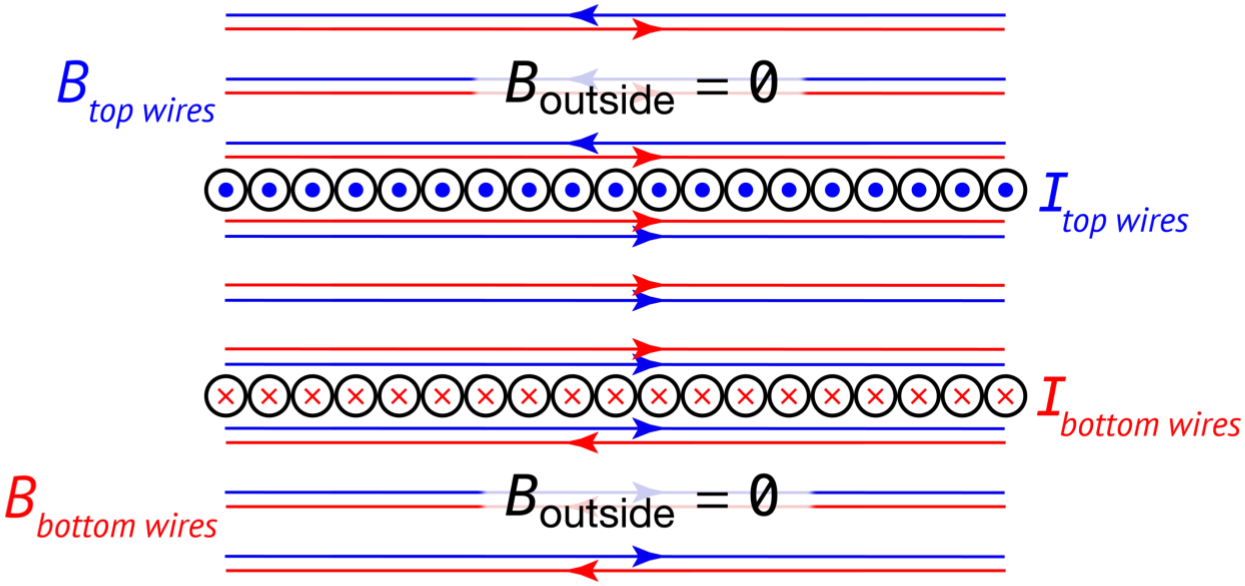
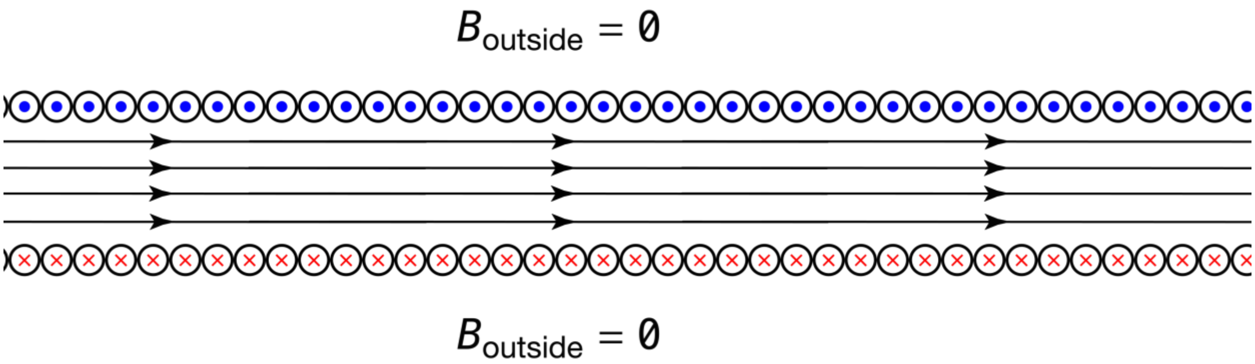
Flipping Physics Lecture Notes:
Solenoid Magnetic Field

http://www.flippingphysics.com/solenoid-b-field.html

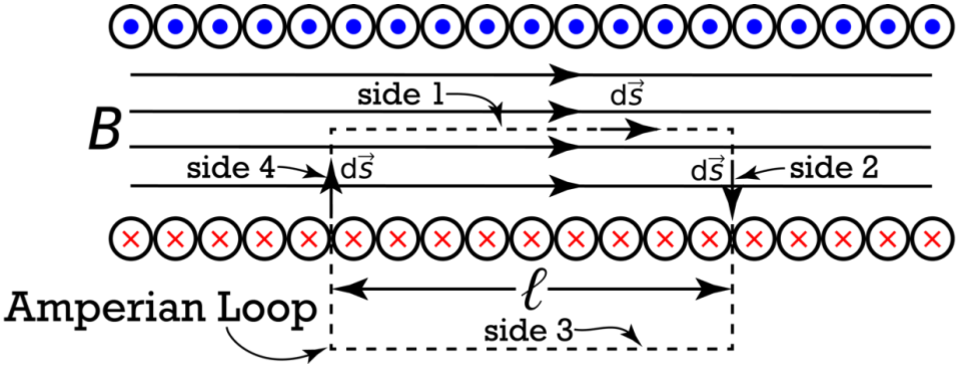
A solenoid is a very common tool for creating a uniform magnetic field. A typical solenoid is a single, very long, current carrying, insulated wire wrapped to form a hollow cylinder. An ideal solenoid has a length which is much, much larger than its diameter. The cross section of a solenoid looks like this.



Outside the solenoid, the magnetic field caused by the current in the top wires completely cancels out the magnetic field caused by the bottom wires. In other words, an ideal solenoid has zero magnetic field outside the cylinder of the solenoid. (ideal solenoid below)



Now let’s derive the equation for the magnetic field inside an ideal solenoid. In order to do so, we begin with Ampère’s law and draw an Amperian loop. Just like Gaussian surfaces, we want to pick Amperian loops to have sides which are at integer multiples of 90° relative to the magnetic field, and such that the magnetic field is uniform on the sides of the Amperian loop. For an ideal solenoid, we pick an Amperian loop shape of a rectangle with one side inside the solenoid and parallel to the magnetic field inside the solenoid and the opposite side of the Amperian loop is completely outside the solenoid.



And now we can begin using Ampère’s law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \Rightarrow \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s} = \mu_0 I_{in}$$

For side 3, the magnetic field is zero outside the solenoid, so that integral equals zero. For sides 2 and 4, the magnetic field and ds are 90° to one another and the cosine of 90° is zero, so both of those integrals equal zero. That means, the only integral which remains is the integral for side 1.

$$\Rightarrow \int_1 \vec{B}\cdot d\vec{s} = B\int_1 ds\cos 0^\circ = B\ell = \mu_0 N I \;\;\&\;\; I_{in} = NI$$

$$\Rightarrow B = \frac{\mu_0 N I}{\ell} \;\;\&\;\; n = \frac{N}{\ell} \Rightarrow B_{solenoid} = \mu_0 n I$$

Where “n” is the turn density of the solenoid.