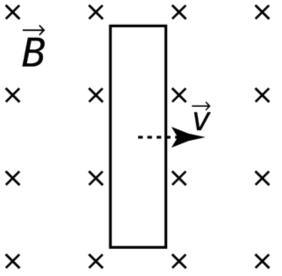
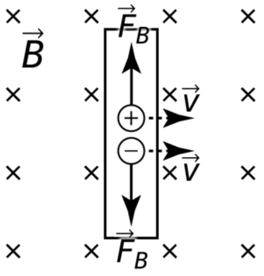
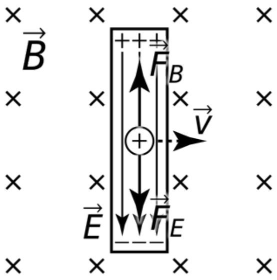
Flipping Physics Lecture Notes:
Induced Forces
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-induced-forces.html

Believe it or not, but our discussion of induced forces begins with two derivations of *motional emf*. Motional emf is the idea that the motion of a conductor moving in a magnetic field can cause charges to move in the conductor creating a voltage across the conductor. In other words, a conductor moving in a magnetic field can acquire an induced emf across it.





- The conductor is moving to the right with a constant velocity at a right angle to a magnetic field which is into the page.
- According to the right-hand rule, positive charges will experience an upward magnetic force, and negative charges will experience a downward magnetic force.
- This will result in the movement of charges with the final result being that there will be a net positive charge on the top end of the conductor and a net negative charge on the bottom end of the conductor. This arrangement of charges creates a uniform, downward electric field in the moving conductor.
- As a result of the downward electric field in the conductor, positive charges will experience a downward electrostatic force, and negative charges will experience an upward electrostatic force.



- Because the conductor is moving at a constant velocity, the charges will arrange themselves such that equilibrium is reached between the magnetic and electric forces acting on the charges such that the electric field has a constant magnitude and the charges in the conductor are moving with a constant velocity to the right; there is no vertical motion of the electric charges.
- We can now sum the forces on a positive charge.
    - The same final equation is derived when using a negative charge.

$$\sum \vec{F}_y = F_B - F_E = ma_y = m(0) = 0 \Rightarrow F_B = F_E \Rightarrow qvB\sin\theta = qE$$

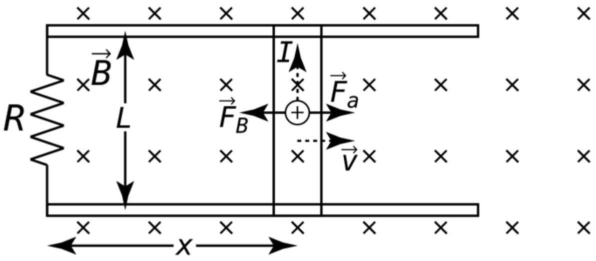
$$\Rightarrow vB\sin 90^\circ = E \Rightarrow vB = E$$

Previously we derived the equation relating voltage and a uniform electric field. We have already identified the direction of the electric field, so we only need the absolute value of the voltage.

$$\& \ \Delta V = -Ed \Rightarrow |\Delta V| = EL \Rightarrow E = \frac{\Delta V}{L} = vB \Rightarrow \Delta V = vBL \Rightarrow \varepsilon = vBL$$

L is the length of the conductor. We have derived the voltage or the induced emf across the conductor moving at a right angle to a uniform magnetic field. This is called *motional emf*.

There is actually an entirely different approach to deriving the same motional emf equation. This approach starts with a conductor moving to the right while in contact with two parallel, metal rails connected by a wire at the left end with a uniform magnetic field going into the page. The resistance of the circuit is represented by the resistor shown in the wire on the left. A force is applied to the conductor to cause it to move to the right. We can use Lenz' law to determine the direction of the induced current in the loop.



- The magnetic field is into the screen and the magnetic flux is increasing because the area of the loop is increasing which increases the number of field lines passing through the loop.

- The induced magnetic field opposes this change in flux and is directed out of the page.
- Using the alternate right-hand rule, our fingers curl in the direction of the induced magnetic field which is out of the page inside the loop and our thumb points in the counterclockwise direction which is in the direction of the induced current in the loop.
- Notice this means that, because positive charges are moving in the direction of conventional current in the conductor, we can use the right-hand rule to show that the fingers point in the direction of the motion of the positive charges which is up, fingers curl in the direction of the magnetic field, which is into the page, and our thumb points in the direction of the magnetic force, which is to the left. In other words, there is a magnetic force which opposes the motion of the conductor in the magnetic field. If the applied force is constant, the magnetic force will also be constant to keep the conductor moving at a constant velocity.
- Now we can use Faraday's law to determine the magnitude of the induced emf in the conductor.

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{dBA\cos\theta}{dt} = -(1)B\cos(180°)\frac{d(Lx)}{dt} = BL\frac{dx}{dt}$$
$$\Rightarrow \varepsilon = vBL$$

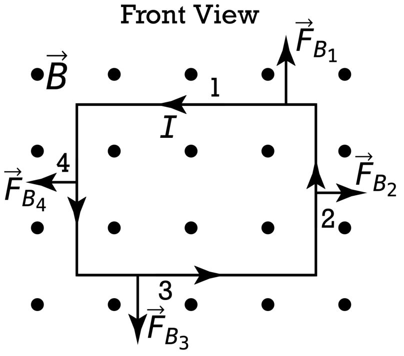
Next, let's look at a conductive loop which has a current induced in it, something we talked about previously[1], that induced current is now a bunch of charge carriers which are moving in a magnetic field. Those moving charges now have induced forces acting on them, again this is something we talked about quite a before now[2]. The following equations determine that magnetic force:

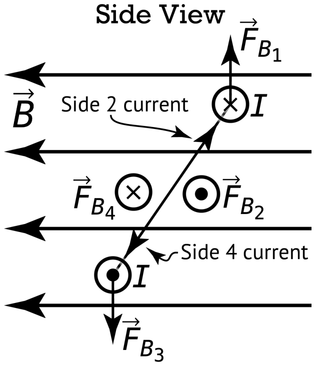
$$\vec{F}_B = I\vec{L} \times \vec{B} \Rightarrow \|F_B\| = ILB\sin\theta$$

Let's walk through an example.

Below is a front view and a side view of a conducting loop in the shape of a rectangle. Let's start by only looking at the front view. Again, all directions for now refer to the *front view only*.

- A uniform magnetic field is directed out of the page and is decreasing.
- That means the magnetic flux through the loop is decreasing.
- Lenz' law states the induced B field is out of the page to counteract the decreasing magnetic flux.
- Using the alternate right-hand rule
    - Fingers curl with the induced B field, out of the page.
    - Thumb points counterclockwise with induced current.
- The right-hand rule on the induced current in side 1 of the loop:
    - Fingers point to the left in the direction of the induced current.
    - Fingers curl out of the page in the direction of the original magnetic field.
    - Thumb points up in the direction of the induced magnetic force on side 1.





- For the remaining sides the right-hand rule shows the induced magnetic forces are:
    - To the right on side 2.
    - Down on side 3.
    - To the left on side 4.
- Notice that the net induced magnetic force on the loop equals zero!
    - The induced magnetic forces on sides 1 and 3 are equal and opposite.
    - The induced magnetic forces on sides 2 and 4 are equal and opposite.

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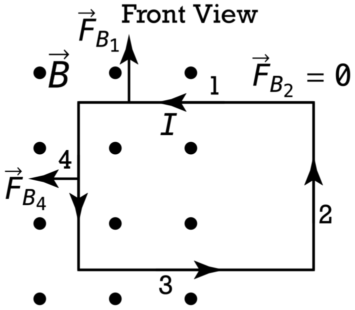
[1] Electromagnetic Induction Review for AP Physics C: E&M- http://www.flippingphysics.com/apcem-electromagnetic-induction.html

[2] Magnetic Fields Review for AP Physics C: E&M - http://www.flippingphysics.com/apcem-magnetic-fields.html

Now let’s switch to the side view. Again, all directions for now refer to the *side view only*.

- Side 1: The induced current is into the page, and the induced magnetic force is up.
- Side 2: The induced current is up and to the right, and the induced B force is out of the page.
- Side 3: The induced current is out of the page, and the induced magnetic force is down.
- Side 4: The induced current is down and to the left, and the induced B force is into the page.
- You can see the net induced magnetic force on the loop is still zero, however, …
- The induced magnetic forces cause a net torque on the loop! Net torque is not zero!
    - Assuming the loop is not attached to anything, the net torque on the loop would cause an angular acceleration around its center of mass which is counterclockwise at this specific moment in time.



Now let’s change the example by making it so the magnetic field abruptly ends partway through the loop.[3]

- Again, the magnetic field is uniform, directed out of the page, and is decreasing in magnitude.
- Lenz’ law gives us the same direction for the induced current in the loop; counterclockwise.
- Using the right-hand rule to determine the directions of the induced magnetic force:
    - Side 2: This entire side of the wire is not in the magnetic field, so there is *no induced magnetic force on side 2!*
    - Side 4: Everything is the same here. Induced magnetic force is to the left.
    - Sides 1 and 3: The directions are the same as before (1 is up, 3 is down), however, only the part of each side which is in the magnetic field will experience an induced magnetic force, therefore, the magnitudes of these induced magnetic forces are smaller than in the previous example.
- The net induced magnetic force on this loop *does not equal zero.*
    - The induced magnetic forces on sides 1 and 3 are equal and opposite.
    - The net force would accelerate the loop to the left.

In other words, the net induced magnetic force on a current carrying loop:

- Which is entirely in a uniform external magnetic field always equals zero.
    - (The induced magnetic forces can cause a net torque on the loop.)
- Which is only partially in a uniform magnetic field is nonzero.
    - This can cause a translational acceleration of the loop.

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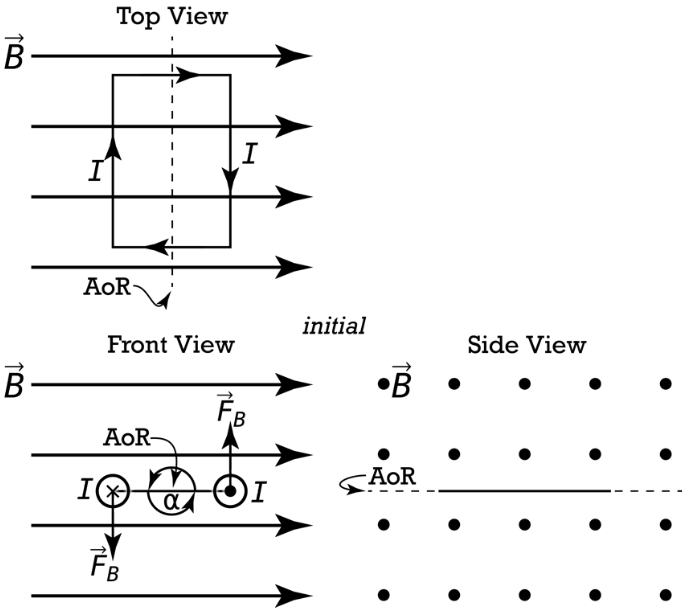
[3] I would argue that creating a magnetic field which looks like this is impossible, however, it is helpful for learning. So, step off!

Let’s switch it up again. In this example we have a rectangular conducting loop in a uniform magnetic field oriented as shown. We place an emf across the loop to cause current I in the loop.

In the *front view* you can see that, according to the right-hand rule, a net torque acts on the loop causing it to angularly accelerate in a clockwise direction (in the front view).



Everything we have been referring to is the initial position of the loop. After the loop has turned 90 degrees, we are now at the final position of the loop.

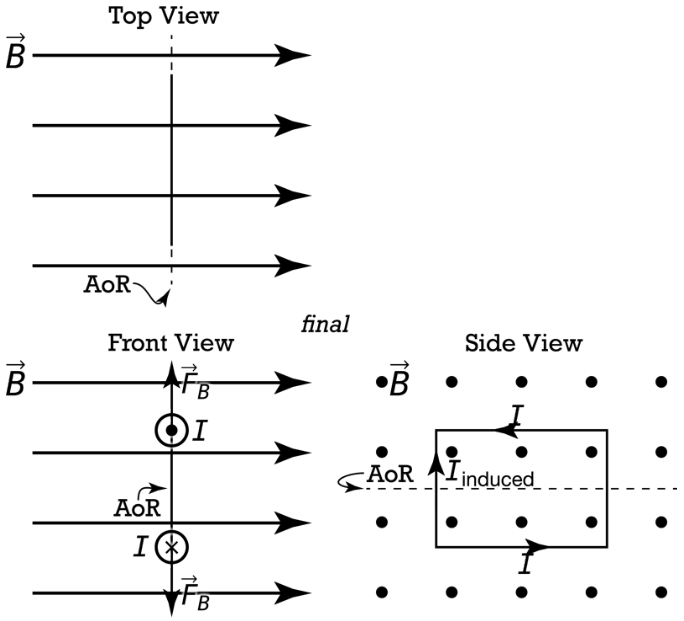
This is a very basic illustration of how an electric motor works. Current is placed through wire loops in magnetic fields which causes the loops to rotate converting electric potential energy to mechanical energy.

Now let’s look at how the magnetic flux changes from the initial to final positions.
The initial magnetic flux through the loop is zero.
The final magnetic flux through the loop is nonzero.
The magnetic flux through the loop changes, which means there is an induced magnetic field, an induced emf, and an induced current in the loop. We need to use Lenz’ law to determine the direction of the induced current.

In the side view, the magnetic flux is out of the page and increasing. In order to resist this change in magnetic flux, the induced magnetic field is into the screen (in the side view). According to the alternate right-hand rule, the fingers curl into the screen in the direction of the induced magnetic field inside the loop, thumb points clockwise (in the side view) in the direction of the induced current in the loop.

In other words, in electric motors, there is an induced emf and an induced current caused by the change in the magnetic flux in the loops of the motor, and that induced current is opposite the direction of the current placed in the loops to cause the loops to rotate. This induced current decreases the current in a turning electric motor. This concept is called *back emf* and is present in all electric motors when they are rotating.

Realize this back emf is not present when the electric motor is not rotating. In other words, when an electric motor is first starting up, the current through the electric motor is larger than when the electric motor is running at a constant angular velocity. This lack of back emf when an electric motor is not moving can cause lights which are on the same circuit to dim when an electric motor is first starting up and can even cause a circuit breaker to trip if something suddenly binds the electric motor causing it to stop rotating which brings the back emf to down zero and suddenly increases the current in the circuit above the maximum current allowed through the circuit breaker.[4]



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[4] Yes, I have done this. ☺