

Flipping Physics Lecture Notes:
Introduction to Power
The rate at which work is done is called Power.

- $\quad P=\frac{W}{\Delta t}$ : Power equals work divided by change in time.
- $\quad P=\frac{W}{\Delta t} \Rightarrow \frac{J}{S}=$ watts : The units for Power are joules per second which we call watts.
$P=\frac{W}{\Delta t}=\frac{F d \cos \theta}{\Delta t}=F\left(\frac{d}{\Delta t}\right) \cos \theta=F v \cos \theta$
- Identify the force(s) delivering the power.
- $\theta$ is the angle between the force and the velocity.
- Use the magnitude of force and velocity in the power equation.
$P=\frac{W}{\Delta t}=F v \cos \theta$ (There are essentially two equations for power.)
Work and change in time are both scalars, so Power is also a scalar.
746 watts $=1.00 \mathrm{hp} \quad \mathrm{hp}=$ horsepower

Why the work is the same in the two examples:
$W_{F_{a}}=F_{a} d \cos \theta$
$d=\Delta y($ same $) \& \theta=0^{\circ}($ same $)$
$\stackrel{\rightharpoonup}{\mathrm{a}}=\frac{\Delta \stackrel{\rightharpoonup}{V}}{\Delta t}=\frac{0}{\Delta t}=0 \& \sum F_{y}=F_{a}-F_{g}=m \vec{a}_{y}=m(0)=0 \Rightarrow F_{a}=F_{g}=m g($ same $)$
Therefore $W_{F_{a}}=F_{a} d \cos \theta($ same $)$


## Flipping Physics Lecture Notes:

## Average and Instantaneous Power Example

Example: An 8.53 kg pumpkin is dropped from a height of 8.91 m . What is the power delivered by the force of gravity (a) over the whole displacement of the pumpkin, (b) right after the pumpkin is dropped and (c) right before the pumpkin strikes the ground?

First we need to understand that part (a) is asking for the average power delivered by the force of gravity because it is the power over a time duration, whereas parts (b) and (c) are asking for instantaneous power because it is at a specific time.
(a) $P=\frac{W}{\Delta t}=F v \cos \theta \Rightarrow P_{F_{g}}=\frac{F_{g} d \cos \theta}{\Delta t}=\frac{(m g) d \cos \theta}{\Delta t}=\frac{(8.53)(9.81)(8.91) \cos (0)}{\Delta t}$

The Force of Gravity and the displacement are both down, so $\theta$, the angle between those two directions, is zero. We need the change in time. The ball is in free fall so ...

Knowns: $\Delta y=-8.91 m ; a_{y}=-g=-9.81 \frac{m}{s^{2}} ; v_{i y}=0 ; \Delta t=$ ?
$\Delta y=v_{i y} \Delta t+\frac{1}{2} a_{y} \Delta t^{2} \Rightarrow-8.91=(0) \Delta t+\frac{1}{2}(-9.81) \Delta t^{2}$
$\Rightarrow \Delta t^{2}=\frac{(-8.91)(2)}{-9.81} \Rightarrow \Delta t=\sqrt{\frac{(8.91)(2)}{9.81}}=1.34778 \mathrm{sec}$
$\& P_{F_{g}}=\frac{(8.53)(9.81)(8.91) \cos (0)}{1.34778}=553.193 \approx 553 \mathrm{watts}$
(Average power delivered by the force of gravity during the entire event.)
Alternate solution: $P_{F_{g}}=F_{F_{g}} v_{\text {avg }} \cos \theta=(\mathrm{mg}) v_{\text {avg }} \cos \theta \& \quad v_{\text {avg }}=\frac{\Delta y}{\Delta t}=\frac{-8.91}{1.34778}=-6.61087 \frac{\mathrm{~m}}{\mathrm{~s}}$ $\& P_{F_{g}}=(m g) v_{\text {avg }} \cos \theta=(8.53)(9.81)(6.61087) \cos (0)=553.193 \approx 553 \mathrm{watts}$
(b) $P_{F_{g}}=\frac{W_{F_{g}}}{\Delta t}=\frac{F_{g} d \cos \theta}{\Delta t}$ Actually, we can't use this equation because the question asks for the instantaneous power and therefore, we need to use the equation for power which has instantaneous velocity in it: $P_{F_{g}}=F_{g} v_{\text {inst }} \cos \theta=F_{g} v_{i y} \cos \theta=F_{g}(0) \cos \theta=0$
(Instantaneous power delivered by the force of gravity at the very start.)
Note: The equation, $P=\frac{W}{\Delta t}$, only works for average power. The equation, $P=F v \cos \theta$, works for both average and instantaneous power.
(c) Again we need to use $P_{F_{g}}=F_{g} V_{\text {inst }} \cos \theta$ because this is instantaneous velocity, however, we need the velocity right before the ball strikes the ground. Again, the ball is in free fall.

$$
\begin{aligned}
& v_{f y}^{2}=v_{i y}^{2}+2 a_{y} \Delta y=0^{2}+(2)(-9.81)(-8.91) \Rightarrow v_{f y}=\sqrt{(2)(-9.81)(-8.91)}=-13.2217 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \& P_{F_{g}}=F_{g} v_{\text {inst }} \cos \theta=(\mathrm{mg}) v_{f y} \cos \theta=(8.53)(9.81)(13.2217) \cos (0)=1106.386 \approx 1110 \mathrm{Watts} \\
& P_{F_{g}}=1106.386 \mathrm{Watts} \times \frac{\mathrm{lhp}}{746 \mathrm{watts}}=1.48369 \approx 1.48 \mathrm{hp} \\
& \text { (Instantaneous power delivered by the force of gravity at the very end.) }
\end{aligned}
$$

Note: Remember in the Work and Power equations, you only use the magnitude of the force, displacement, and velocity.


Flipping Physics Lecture Notes:
Graphing Instantaneous Power
Example: An 8.53 kg pumpkin is dropped from a height of 8.91 m . Will the graph of instantaneous power delivered by the force of gravity as a function of $\qquad$ be linear? If not, what would you change to make the graph linear? (a) Time, (b) Position.

The equation for instantaneous power delivered by the force of gravity is:
$P_{F_{g}}=F_{g} V_{\text {inst }} \cos \theta=(m g) V_{\text {inst }} \cos (0)=m g v_{\text {inst }}$
We can substitute mass times the acceleration due to gravity because that is the equation for the force of gravity. And we can substitute zero degrees for theta because the force of gravity is down and the instantaneous velocity is down and the angle between down and down is zero degrees.

What we need is an expression for the instantaneous velocity. The pumpkin is in free fall so it's acceleration equals -g or $-9.81 \mathrm{~m} / \mathrm{s}^{2}$. Because the acceleration is constant, we can use the uniformly accelerated motion (UAM) equations. Which means the instantaneous velocity we are referring to here is the final velocity in the $y$-direction in the UAM equations.

Let's start with part (a): Instantaneous Power as a function of time. Therefore, we need the velocity final in the $y$-direction of the pumpkin in terms of time:
$v_{\text {fy }}=v_{i y}+a_{y} \Delta t=0+(-g)\left(t_{f}-t_{i}\right)=(-g)\left(t_{f}-0\right)=-g t_{f}$
Before we plug -gtf into the equation for instantaneous power, recall that we use only the magnitude of the force and the velocity in the power equation, therefore, we use +gt instead.
$P_{F_{g}}=m g\left(g t_{f}\right)=m g^{2} t_{f} \& y=($ slope $) x+b$
(I use this instead of $y=m x+b$ to avoid redundant " $m$ " variables.)
Note: With Power on the $y$-axis and time final on the $x$-axis, we should get a linear relationship with $\mathrm{mg}^{2}$ as the slope of the line and a y-intercept of zero.

So the answer to part (a) is "Yes, Instantaneous Power as a function of Time will be linear."


$$
\text { slope }=m g^{2}=(8.53)\left(9.81^{2}\right)=820.894 \approx 821 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{4}}
$$

Part (b) Instantaneous Power as a function of height. Therefore, we need the velocity final in the ydirection of the pumpkin in terms of position:
${v_{f y}}^{2}={v_{i y}}^{2}+2 a_{y} \Delta y=0^{2}+2(-g)\left(y_{f}-y_{i}\right)=-2 g\left(y_{f}-0\right) \Rightarrow v_{f y}=\sqrt{-2 g y_{f}}$
Note: We have set the initial position to be zero.
Which we can plug back into the equation for instantaneous power:
$P_{F_{g}}=m g v_{\text {inst }}=m g \sqrt{-2 g y_{f}} \& y=($ slope $) x+b$
(arggg!! Two different $y$ 's. Note: " $y_{f}^{\prime}$ " is the final position of the pumpkin and " $y$ " represents the $y$-axis variable.)
Notice that, because of the square root in the equation, this will not yield a linear relationship between $P_{F_{g}}$ and $\mathrm{y}_{\mathrm{f}}$. The answer to part (b) is "No, Instantaneous Power as a function of Height will not be linear." And we need to square the equation to determine what to put on the $x$ and $y$ axis to get a linear relationship.
$\left(P_{F_{g}}\right)^{2}=m^{2} g^{2}\left(-2 g y_{f}\right)=-2 m^{2} g^{3} y_{f} \& y=($ slope $) x+b$
Therefore power squared is on the $y$-axis and position is on the $x$-axis:


$$
\text { slope }=-2 \mathrm{~m}^{2} g^{3}=-(2)\left(8.53^{2}\right)\left(9.81^{3}\right)=-137384 \approx-137000 \frac{\mathrm{~kg}^{2} \cdot \mathrm{~m}^{3}}{\mathrm{~s}^{6}}
$$



## Flipping Physics Lecture Notes:

## Average Power Delivered by a Car Engine - Example Problem

Example: A 1400 kg Prius uniformly accelerates from rest to $3.0 \times 10^{1} \mathrm{~km} / \mathrm{hr}$ in 9.25 seconds and 42 meters. If an average force of drag of 8.0 N acts on the car, what is the average power developed by the engine in horsepower?

Knowns:

$$
\begin{aligned}
& m=1400 \mathrm{~kg} ; v_{i}=0 ; \Delta t=9.25 \mathrm{sec} ; \Delta x=42 \mathrm{~m} ; v_{f}=30 \frac{\mathrm{~km}}{\mathrm{hr}} \times \frac{\mathrm{lhr}}{3600 \mathrm{sec}} \times \frac{1000 \mathrm{~m}}{\mathrm{lkm}}=8 . \overline{3} \frac{\mathrm{~m}}{\mathrm{~s}} \\
& F_{\text {Drag }}=8.0 \mathrm{~N} ; P_{\text {average by engine }}=? \& P=\frac{W}{\Delta t}=F v \cos \theta
\end{aligned}
$$

We can use either of these equations because we are solving for average power. If the question asked for instantaneous power at a particular point along the trip, then we would need to use $P=F v \cos \theta$ and use instantaneous velocity in the equation.

Average power delivered by a car engine. Let's illustrate that using a force applied to the car.
And let's use the equation $P_{F_{a}}=\frac{W_{F_{a}}}{\Delta t}=\frac{F_{a} d \cos \theta}{\Delta t}$
Draw Free Body Diagram:

$\sum F_{x}=F_{a}-F_{D r a g}=m a_{x} \Rightarrow F_{a}=m a_{x}+F_{D r a g} \quad$ We need $\mathrm{a}_{\mathrm{x}}$ to solve for $\mathrm{F}_{\mathrm{a}}$.
$a_{x}=\frac{\Delta v}{\Delta t}=\frac{v_{f}-v_{i}}{\Delta t}=\frac{8 . \overline{3}-0}{9.25}=0.9009 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$F_{a}=m a_{x}+F_{D r a g}=(1400)(0.9009)+8.0=1269.26 \mathrm{~N}$
The force applied is to the right and the displacement is to the right, therefore the angle, $\theta$, in the work equation is zero degrees. $\theta=0^{\circ}$.
$P_{F_{a}}=\frac{W_{F_{a}}}{\Delta t}=\frac{F_{a} d \cos \theta}{\Delta t}=\frac{(1269.26)(42) \cos (0)}{9.25}=5763.13 \mathrm{watts}$
$P_{F_{a}}=5763.13$ watts $\times \frac{\mathrm{lhp}}{746 \text { watts }}=7.7256 \approx 7.7 \mathrm{hp}$

Larger Acceleration Example:
$m=1400 \mathrm{~kg} ; v_{i}=0 ; \Delta t=5.43 \mathrm{sec} ; ; \Delta x=42 \mathrm{~m} ; v_{f}=56 \frac{\mathrm{~km}}{\mathrm{hr}} \times \frac{\mathrm{lhr}}{3600 \mathrm{sec}} \times \frac{1000 \mathrm{~m}}{1 \mathrm{~km}}=15 . \overline{5} \frac{\mathrm{~m}}{\mathrm{~s}}$ $V_{f}=56 \frac{\mathrm{~km}}{\mathrm{hr}} \times \frac{\mathrm{mi}}{1.609 \mathrm{~km}}=34.804 \approx 35 \frac{\mathrm{mi}}{\mathrm{hr}} ; F_{\text {Drag }}=26 \mathrm{~N} ; P_{\text {average by engine }}=$ ?
$a_{x}=\frac{\Delta v}{\Delta t}=\frac{v_{f}-v_{i}}{\Delta t}=\frac{15 . \overline{5}-0}{5.43}=2.86474 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \& F_{a}=m a_{x}+F_{\text {Drag }}=(1400)(2.86474)+26=4036.64 \mathrm{~N}$ $W_{F_{a}}=F_{a} d \cos \theta=(4036.64)(42) \cos (0)=169539 J \approx 170000 J$ (Larger Acceleration Example)
$W_{F_{a}}=F_{a} d \cos \theta=(1269.26)(42) \cos (0)=53309 \mathrm{~J} \approx 53000 \mathrm{~J}$ (Original, Smaller Acceleration Example)
$\& \frac{169539-53309}{53309} \times 100=218.03 \approx 220 \% \& \frac{5.43-9.25}{9.25} \times 100=-41.297 \approx-41 \%$
In other words, in order to decrease the time of the event by $41 \%$, you need to use $220 \%$ more energy. That's a lot of unnecessary horses, hay and poop on the road.


Flipping Physics Lecture Notes:
Solving for Force of Drag on an Accelerating Car
In my previous video "Average Power Delivered by a Car Engine" there was an "average drag force of 8.0 N " acting on a Prius. Here is how I solved for that number. Previous Video: http://www.flippingphysics.com/average-power.html
A standard equation for the Force of Drag is $F_{D r a g}=\frac{1}{2} \rho v^{2} D A$.

- $\quad \rho$, is the density of the medium through which the object is moving. In our example, the density of air. In order to know the density of air, we need the air temperature, which was $67^{\circ} \mathrm{F}$.

$$
T_{e m p}^{a i r}=67^{\circ} F \Rightarrow T_{{ }^{\circ} C}=\left(T_{{ }^{\circ} F}-32\right)\left(\frac{5}{9}\right)=(67-32)\left(\frac{5}{9}\right)=19 . \overline{4} \approx 19^{\circ} \mathrm{C}
$$

- According to The Engineering Toolbox ${ }^{1}, 1.2 \mathrm{~kg} / \mathrm{m}^{3}$ is a good approximation for the density of air at this temperature.
- $\quad D$, is the Drag Coefficient of the object, in this case the Prius. According to EcoModder ${ }^{2}$, the Drag Coefficient of my 2011 Toyota Prius is 0.25 .
- $\quad A$, is the Cross Sectional Area perpendicular to the direction of motion. This is the Frontal Area listed on the same EcoModder page. It is 21.6 square feet. Of course, we need it in square meters.

$$
\quad A=21.6 f t^{2} \times\left(\frac{1 m}{3.281 f t}\right)^{2}=2.00651 m^{2}
$$

- $\quad \boldsymbol{V}$, is the velocity of the car. Unfortunately, because the velocity is squared in the equation for the Force of Drag, we cannot simply find the average velocity and use that. Instead, we need to use the instantaneous velocity of the car to plot the instantaneous force of drag as a function of time and use an integral.

| Time <br> $(\mathrm{s})$ | Interval <br> Time (s) | Instantaneous <br> Speed (km/hr) | Instantaneous <br> Speed (m/s) | Instantaneous <br> Force of Drag (N) |
| :---: | :---: | :---: | :---: | :---: |
| 0.00 |  | 0 | 0.00 | 0.00 |
| 1.10 | 1.10 | 2 | 0.56 | 0.093 |
| 1.38 | 0.28 | 3 | 0.83 | 0.21 |
| 1.67 | 0.29 | 4 | 1.11 | 0.37 |
| 1.95 | 0.28 | 6 | 1.67 | 0.84 |
| 2.22 | 0.27 | 7 | 1.94 | 1.14 |
| 2.50 | 0.28 | 8 | 2.22 | 1.49 |
| 2.78 | 0.28 | 10 | 2.78 | 2.32 |
| 3.07 | 0.29 | 11 | 3.06 | 2.81 |
| 3.35 | 0.28 | 12 | 3.33 | 3.34 |
| 3.62 | 0.27 | 13 | 3.61 | 3.92 |
| 3.90 | 0.28 | 14 | 3.89 | 4.55 |
| 4.18 | 0.28 | 15 | 4.17 | 5.23 |
| 4.47 | 0.29 | 16 | 4.44 | 5.95 |
| 4.75 | 0.28 | 17 | 4.72 | 6.71 |
| 5.02 | 0.27 | 18 | 5.00 | 7.52 |
| 5.30 | 0.28 | 19 | 5.28 | 8.38 |

[^0]

Here is what I did to create the table and graph above:

- Use the video to determine the time when the speedometer reading changed. This is the "Time" column.
- Determine the interval for each change in the speedometer reading. This is the "Interval Time" column.
- Notice it appears the Prius' speedometer updates slightly more than three times every second.
- The "Instantaneous Speed" is the speedometer reading. Originally this is in kilometers per hour.
- Convert "Instantaneous Speed" to meters per second.
- Last reading: Instantaneous Speed $=30 \frac{\mathrm{~km}}{\mathrm{hr}} \times \frac{\mathrm{lhr}}{3600 \mathrm{sec}} \times \frac{1000 \mathrm{~m}}{1 \mathrm{~km}}=8 . \overline{3} \frac{\mathrm{~m}}{\mathrm{~s}}$
- Determine the "Instantaneous Force of Drag". This is the force of drag at each time and it is "instantaneous" because it uses the velocity at that specific point in time.
- Last reading: $F_{D \text { rag }}=\frac{1}{2} \rho v^{2} D A=\frac{1}{2}(1.2)(8 . \overline{3})^{2}(0.25)(2.00651)=20.90 N$
- Plot all of the data: Force of Drag as a function of Time. Note the "R squared value" of 0.999 is very close to 1. A value of 1 would be a $100 \%$ perfect fit, our best-fit curve is $99.9 \%$ accurate, which is quite good.
- Add a best-fit curve. Excel reports $y=0.2275 x^{2}+0.3187 x$ as the best-fit curve. However, we know Force of Drag is on the $y$-axis and time is on the x-axis. Therefore, the best-fit curve equation actually is

$$
F_{D r a g}=0.2275 t^{2}+0.3187 t
$$

- Take the definite integral of the Force of Drag with respect to time to get the area under the curve.

$$
\begin{aligned}
& \circ \int_{0}^{9.25} F_{\text {Drag }} d t=\int_{0}^{9.25}\left(0.2275 t^{2}+0.3187 t\right) d t=\left[\frac{0.2275 t^{3}}{3}+\frac{0.3187 t^{2}}{2}\right]_{0}^{9.25} \\
\Rightarrow & \int_{0}^{9.25} F_{D r a g} d t=\frac{0.2275(9.25)^{3}}{3}+\frac{0.3187(9.25)^{2}}{2}-\left(\frac{0.2275(0)^{3}}{3}+\frac{0.3187(0)^{2}}{2}\right)^{2}=73.653 \mathrm{~N} \cdot \mathrm{~S}
\end{aligned}
$$

- This is also equal to the Average Force of Drag times the Change In Time. So we can solve for the Average Force of Drag.

$$
\int_{0}^{9.25} F_{\text {Drag }} d t=F_{\text {Drag Average }} \Delta t=73.653 \Rightarrow F_{\text {Drag Average }}=\frac{73.653}{\Delta t}=\frac{73.653}{9.25}=7.962 \approx 8.0 \mathrm{~N}
$$

Now, I am sure some of you are wondering why we can't just use the average velocity:

$$
v_{\text {average }}=\frac{\Delta x}{\Delta t}=\frac{42}{9.25}=4.541 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

And then solve for the Average Force of Drag.

$$
F_{D \text { rag }}=\frac{1}{2} \rho v^{2} D A=\frac{1}{2}(1.2)(4.541)^{2}(0.25)(2.00651)=6.206 \approx 6.2 N \text { (incorrect) }
$$

This incorrect solution does not account for the fact that the velocity is squared in the force of drag equation and therefore the instantaneous force of drag does not increase linearly. Using the average velocity to solve for the average force of drag would only be correct if the force of drag increased linearly. The faster the car moves, the more incorrect this incorrect solution becomes. See below:

Note: The more chunks of time we can break the event into, the more accurate the calculation for the curve. In this particular case we were limited by the refresh rate of the speedometer.

Graph for the second car:


$$
\begin{aligned}
& \int_{0}^{5.43} F_{D r a g} d t=\int_{0}^{5.43}\left(2.8667 t^{2}-0.6267\right) d t=\left[\frac{2.8667 t^{3}}{3}-\frac{0.6267 t^{2}}{2}\right]_{0}^{5.43} \\
& \Rightarrow \int_{0}^{5.43} F_{\text {Drag }} d t=\frac{2.8667(5.43)^{3}}{3}-\frac{0.6267(5.43)^{2}}{2}-\left(\frac{2.8667(0)^{3}}{3}-\frac{0.6267(0)^{2}}{2}\right)=143.750 \mathrm{~N} \cdot \mathrm{~S} \\
& \int_{0}^{6.98} F_{\text {Drag }} d t=F_{\text {Drag Average }} \Delta t=143.75 \Rightarrow F_{\text {Drag Average }}=\frac{143.75}{\Delta t}=\frac{143.75}{5.43}=26.473 \approx 26 \mathrm{~N}
\end{aligned}
$$

Incorrect Solution:
$V_{\text {average }}=\frac{\Delta x}{\Delta t}=\frac{42}{5.43}=7.735 \frac{\mathrm{~m}}{\mathrm{~s}}$
$F_{\text {Drag }}=\frac{1}{2} \rho v^{2} D A=\frac{1}{2}(1.2)(7.735)^{2}(0.25)(2.00651)=18.007 \approx 18 N$
See, it's more incorrect the faster the vehicle moves.


Flipping Physics Lecture Notes:
Instantaneous Power Delivered by a Car Engine - Example Problem
Example: A Toyota Prius is traveling at a constant velocity of $113 \mathrm{~km} / \mathrm{hr}$. If an average force of drag of 3.0 $x 10^{2} \mathrm{~N}$ acts on the car, what is the power developed by the engine in horsepower?

Knowns: $v=113 \frac{\mathrm{~km}}{\mathrm{hr}} \times \frac{1000 \mathrm{~m}}{1 \mathrm{~km}} \times \frac{\mathrm{lhr}}{3600 \mathrm{sec}}=31.3 \overline{8} \frac{\mathrm{~m}}{\mathrm{~s}} ; F_{\text {Drag }}=300 \mathrm{~N} ; P_{\text {by engine }}=?$ $P=\frac{W}{\Delta t}=F v \cos \theta$
We can't use $P=\frac{W}{\Delta t}$ because we are solving for instantaneous power. We need to use $P_{F_{a}}=F_{a} v \cos \theta$ and use instantaneous velocity in the equation. We need $\mathrm{F}_{\mathrm{a}}$, and $\theta$.

Draw Free Body Diagram:
$\sum F_{x}=F_{a}-F_{D r a g}=m a_{x}=m(0)=0 \Rightarrow F_{a}=F_{D r a g}=300 \mathrm{~N}$

The force applied is to the right and the displacement is to the right, therefore the angle, $\theta$, in the work equation is zero degrees. $\theta=0^{\circ}$.

$P_{F_{a}}=F_{a} v \cos \theta=(300)(31.3 \overline{8}) \cos (0)=9416 . \overline{6} \mathrm{watts} \times \frac{1 \mathrm{hp}}{746 \mathrm{watts}}=12.623 \approx 13 \mathrm{hp}$

Note: At 129 kilometers per hour ...

$$
\begin{aligned}
& V=129 \frac{\mathrm{~km}}{\mathrm{hr}} \times \frac{1 \mathrm{mi}}{1.609 \mathrm{~km}}=80.174 \approx 80.2 \frac{\mathrm{mi}}{\mathrm{hr}} \& v=129 \frac{\mathrm{~km}}{\mathrm{hr}} \times \frac{1000 \mathrm{~m}}{\mathrm{lkm}} \times \frac{\mathrm{lhr}}{3600 \mathrm{sec}}=35.8 \overline{3} \frac{\mathrm{~m}}{\mathrm{~s}} \\
& P_{F_{a}}=F_{a} v \cos \theta=(390)(35.8 \overline{3}) \cos (0)=13975 \mathrm{watts} \times \frac{\mathrm{lhp}}{746 \mathrm{watts}}=18.733 \approx 19 \mathrm{hp} \\
& \frac{129 \frac{\mathrm{~km}}{\mathrm{hr}}-113 \frac{\mathrm{~km}}{\mathrm{hr}}}{113 \frac{\mathrm{~km}}{\mathrm{hr}}} \times 100=14.159 \approx 14.2 \% \& \frac{18.733 \mathrm{hp}-12.623 \mathrm{hp}}{12.623 \mathrm{hp}} \times 100=48.406 \approx 48 \%
\end{aligned}
$$

In other words, in order to go 14\% faster, the car consumes 48\% more energy every second.

## Force of Drag Calculation:

A standard equation for the Force of Drag is $F_{D r a g}=\frac{1}{2} \rho v^{2} D A$.

- $\quad \rho$, is the density of the medium through which the object is moving. In our example, the density of air. In order to know the density of air, we need the air temperature, which was $72^{\circ} \mathrm{F}$.
- $T e m p_{\text {air }}=71^{\circ} F \Rightarrow T_{{ }^{\circ} \mathrm{C}}=\left(T_{{ }_{\circ} F}-32\right)\left(\frac{5}{9}\right)=(71-32)\left(\frac{5}{9}\right)=21 . \overline{6} \approx 22^{\circ} \mathrm{C}$
- According to The Engineering Toolbox ${ }^{1}, 1.2 \mathrm{~kg} / \mathrm{m}^{3}$ is a good approximation for the density of air at this temperature.
- $\quad V$, is the instantaneous velocity of the car, $31.3 \overline{8} \frac{m}{\mathrm{~s}}$.
- $D$, is the Drag Coefficient of the object, in this case the Sienna. According to EcoModder ${ }^{2}$, the Drag Coefficient of my 2011 Toyota Prius is 0.25 .
- $\quad A$, is the Cross Sectional Area perpendicular to the direction of motion. This is the Frontal Area listed on the same EcoModder page. It is 21.6 square feet. Of course, we need it in square meters.

$$
\begin{aligned}
& A=21.6 f f^{2} \times\left(\frac{1 \mathrm{~m}}{3.281 \mathrm{ft}}\right)^{2}=2.00651 \mathrm{~m}^{2} \\
& F_{\text {Drag }}=\frac{1}{2} \rho v^{2} D A=\frac{1}{2}(1.2)(31.3 \overline{8})^{2}(0.25)(2.00651)=296.541 \approx 3.0 \times 10^{2} \mathrm{~N} \quad @\left(113 \frac{\mathrm{~km}}{\mathrm{hr}}\right) \\
& F_{D \text { rag }}=\frac{1}{2} \rho v^{2} D A=\frac{1}{2}(1.2)(35.8 \overline{3})^{2}(0.25)(2.00651)=386.462 \approx 390 \mathrm{~N} \quad @\left(129 \frac{\mathrm{~km}}{\mathrm{hr}}\right)
\end{aligned}
$$

For those of you who watched the average power video (http://www.flippingphysics.com/averagepower.html), which used the same Prius, you might be wondering why that example had such a comparatively small force of drag at 8.0 N . The force is so much larger in this problem because the speed of the car is so much larger in this problem. The average speed in the average power problem was roughly $5 \mathrm{~m} / \mathrm{s}$, rather than roughly $31 \mathrm{~m} / \mathrm{s}$ in this problem. Remember the speed is squared in the force of drag equation, which is why the force of drag is so much larger in this problem.

[^1]
[^0]:    ${ }^{1} \mathrm{http}: / / \mathrm{www} . e n g i n e e r i n g t o o l b o x . c o m / a i r-d e n s i t y-s p e c i f i c-w e i g h t-d \_600 . h t m l ~$
    ${ }^{2}$ http://ecomodder.com/wiki/index.php/Vehicle_Coefficient_of_Drag_List

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