Flipping Physics Lecture Notes:
Buoyant Force Explained: Submerged Objects in Fluids
http://www.flippingphysics.com/buoyant-force.html

In our previous lesson we derived the Buoyant Force, the upward force acting on an object which is in a fluid whose magnitude equals the weight of the fluid displaced by the object.

$$F_{\text{Buoyant}} = m_{\text{fluid displaced}} g \Rightarrow F_B = m_f g$$

The Buoyant Force is often called Archimedes' Principle, because it was discovered by the Greek physician, engineer, and mathematician Archimedes.

Often this equation is used in terms of density and volume instead of mass. We know the equation for density is density equals mass divided by volume, which we can rearrange to get mass equals density times volume.

$$\rho = \frac{m}{V} \Rightarrow m = \rho V$$

Which we can substitute back into the equation for Buoyant Force.

$$F_B = m_f g \Rightarrow F_B = \rho_f V_f g$$

Let's look at 3 examples of different objects submerged in water. But first realize, when an object is submerged[1] in a fluid, the volume of the fluid displaced by the object is the same as the volume of the object: When object is submerged: $V_f = V_o$

1) A wood sphere.
    a. The density of wood is less than the density of water.
    b. The density of the object is less than the density of the fluid it is displacing.
    c. The magnitude of the upward Buoyant Force acting on the object is greater than the magnitude of the downward force of gravity acting on the object.
        i. When released, the object will accelerate upward.

$$\sum F_y = F_B - F_g = m_o a_y \ \& \ F_B = m_f g = \rho_f V_f g \ \& \ F_g = m_o g = \rho_o V_o g$$
$$\rho_o < \rho_f \Rightarrow F_B > F_g \Rightarrow a_y > 0$$

2) A rubber sphere.
    a. The density of this rubber is more than the density of water.
    b. The density of the object is more than the density of the fluid it is displacing.
    c. The magnitude of the upward Buoyant Force acting on the object is less than the magnitude of the downward force of gravity acting on the object.
        i. When released, the object will accelerate downward.

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[1] Please realize that adding the word "completely" to submerged is not necessary. The word "submerged" means to be completely surrounded by water. So, yeah, I will do my best to not say "completely submerged" and instead just say "submerged". [I'd add a footnote to this footnote if I could because "completely" is also unnecessary on "completely surrounded", but, I'll stop here.]

$$\rho_o > \rho_f \Rightarrow F_B < F_g \Rightarrow a_y < 0$$

3) A water balloon.
    a. The density of water balloon is the same as the density of water.
    b. The density of the object is the same as the density of the fluid it is displacing.
    c. The magnitude of the upward Buoyant Force acting on the object is the same as the magnitude of the downward force of gravity acting on the object.
        i. When released, the object will have zero acceleration and float in the water.[2]

$$\rho_o = \rho_f \Rightarrow F_B = F_g \Rightarrow a_y = 0$$

We can see that it is only the relative densities of the object and fluid displaced by the object which determine the direction of the acceleration of an object submerged in a fluid.

- $\rho_o < \rho_f \Rightarrow a_y > 0$
- $\rho_o > \rho_f \Rightarrow a_y < 0$
- $\rho_o = \rho_f \Rightarrow a_y = 0$

But how does a steel boat float? The air inside the hull of the boat has such a low density that it makes it so the average density of the boat is lower than the density of water, which makes it possible for steel boats to float.

Notice the Buoyant Force is independent of depth in the fluid. Typically, we deal with ideal fluids which are incompressible. That means the density of the fluid will not change with depth. This is similar to assuming the acceleration due to gravity is constant near the surface of the planet. It's not quite true, however, under normal circumstances the error is negligible.

[2] I regret to inform you that I did have to add a little salt to the water inside the water balloon. This is because, no matter how hard I tried, there were little pockets of air trapped inside the water balloon which decreased the average density of the water balloon. Saltwater has slightly higher density than water. By adding saltwater to the balloon, I was able to achieve a water balloon with the same density as water.