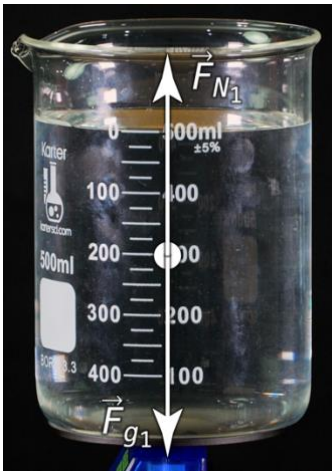
Flipping Physics Lecture Notes:
Buoyant Force Demonstration with Steel and Oak Spheres
http://www.flippingphysics.com/buoyant-force-spheres.html



Let's begin by filling a beaker with water to the zero marker on the beaker and drawing the free body diagram of all the forces acting on the beaker:

- Force of gravity on the water and beaker, down.
  - I've added a "1" to this subscript because there will be a 2nd force of gravity on the water and beaker later.
  - Normally I would not combine two forces of gravity like this, however, there are going to be many forces in these free body diagrams and reducing that number by one will be helpful for your learning.
- Force normal on the beaker, up.
  - I've added a "1" to this subscript because there could be a 2nd normal force on the beaker later.

And sum the forces in the y-direction:

$$\sum F_y = F_{N_1} - F_{g_1} = ma_y = m(0) = 0 \Rightarrow F_{N_1} = F_{g_1}$$

Now we zero the scale holding the beaker and water, identifying this as the "zero" reference point for the system.

Remember that we showed a digital scale measures the force normal caused by the scale not the force of gravity (or weight) of anything.

- If I push down on the beaker, the scale measurement goes up because I am applying a downward force on the beaker and the upward normal force needs to increase to keep the beaker from accelerating in the y-direction.
  - $\sum F_y = F_N - F_{g_1} - F_a = ma_y = m(0) = 0 \Rightarrow F_N = F_{g_1} + F_a$
- If I pull up on the beaker, the scale measurement goes down because I am applying an upward force on the beaker and the upward normal force needs to decrease to keep the beaker from accelerating in the y-direction.
  - $\sum F_y = F_N - F_{g_1} + F_a = ma_y = m(0) = 0 \Rightarrow F_N = F_{g_1} - F_a$

Now we empty the beaker of water, hang a steel sphere in the beaker, and fill the beaker with water to the same zero marker. Even though there is less water in the beaker, the reading on the scale again reads zero because the upward buoyant force on the steel sphere has a magnitude equal to the weight, or force of gravity, of the water displaced by the steel sphere. That upward buoyant force is one of the forces in a Newton's Third Law force pair: $\vec{F}_{\text{water}\rightarrow\text{steel}} = -\vec{F}_{\text{steel}\rightarrow\text{water}}$

- Buoyant force up from water on steel sphere.
- Buoyant force down from steel sphere on water.
  - These are equal in magnitude and opposite in direction.

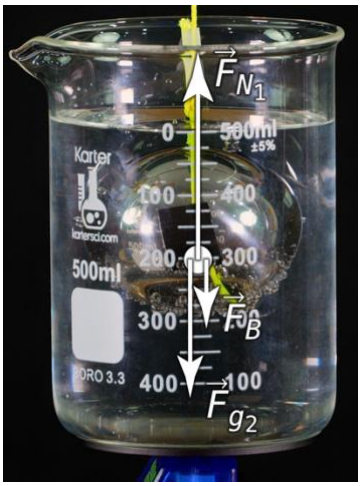
Because there is less water in the beaker, the second force of gravity acting down on the beaker and water is less than before: $F_{g_2} < F_{g_1}$

With the steel sphere in the water, the downward force of gravity on the beaker and on the water in the beaker, F_{g2} , plus the downward force of gravity on the water displaced by the steel sphere, F_B , are equal in magnitude to the force of gravity of the beaker and the water in the beaker without the steel sphere F_{g1} .

$$F_{g_2} + F_B = F_{g_1}$$

If you would like to understand more about this, I have a video where we discuss this in more detail.

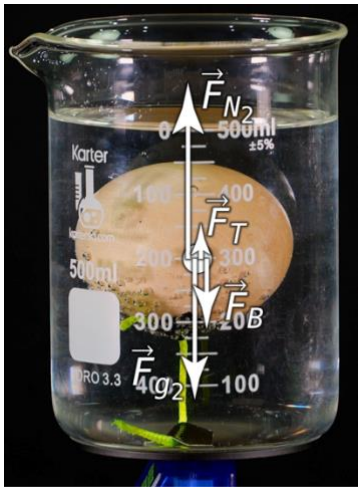
And the question we are here to ask is:
"When we attach an oak sphere with the same diameter as the steel sphere to the bottom of the beaker using a string and add water to the same zero marker as in the previous 2 examples, will the reading on the scale be positive, negative, or zero?"



Take a moment to look at the free body diagram of the forces acting on the beaker with the steel sphere hanging in the water:

- Force of gravity on water and beaker, down.
  - Now with a "2" subscript because it is reduced from before because there is less water in the beaker.
- Buoyant force, down.
  - As described earlier.
- Force normal, up.
  - Has the same magnitude as with just the water in the beaker, F_{N1} . Again, that is why the scale reading is still zero, the same as it was with just water filled to the 0 line on the beaker.

Now with the oak sphere submerged in the water and held down to the bottom of the beaker using the string, let's draw the free body diagram of all the forces acting on the beaker:



- Force of gravity on water and beaker, down.
  - Same "2" subscript because there is the same amount of water in the beaker with the oak sphere and the steel sphere.
- Buoyant force, down.
  - Same as before.
- Force of tension, up.
  - This is caused by the string pulling up on the beaker.
  - This is the only new force in the free body diagram, therefore…
- Force normal, up.

- Now with a subscript of “2” because it is different than before because of the presence of the upward tension force.
- Force normal 2 is less than force normal 1 because that upward force of tension decreases the force normal, just like when I lifted the beaker earlier.

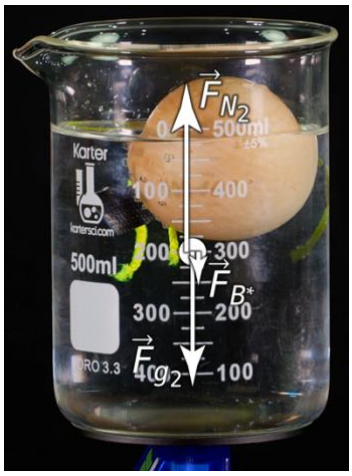
Because the force normal is decreased by the upward tension force, $F_{N_2} < F_{N_1}$, the reading on the scale goes down from zero and is negative. And, the reading on the scale of -0.23 N confirms our answer.

Notice what happens when the string is no longer attached to the bottom of the beaker.

- The oak sphere floats to the top of the water.
- The reading on the scale stays at -0.23 N.

The new free body diagram:

- Force of gravity on water and beaker, down.
  - Same “2” subscript because the amount of water in the beaker is unchanged.
- There is no longer a force of tension.
- Buoyant force, down. F_{B^*}
  - The buoyant force is now reduced because the oak sphere is no longer submerged in the water. The oak sphere displaces less water, reducing the magnitude of the buoyant force.
- Force normal, up.
  - The same “2” subscript, because it is the same as when the oak sphere was attached to the bottom of the beaker using the string.



And then when I push down on the oak sphere, the scale returns to zero because, as far as the scale is concerned, the free body diagram is the same as when the steel sphere was submerged in the water. This is because the steel sphere and the oak sphere have the same volume and displace the same weight in water and therefore have the same buoyant force.