Introductory Concepts:

- **Vector**: Magnitude and Direction
  - Magnitude means the “amount” of the vector or the value of the vector without direction.
- **Scalar**: Magnitude only, no direction
- **Component Vectors**
  - Theta won’t always be with the horizontal, so the component in the x direction won’t always use cosine.
  - \( \sin \theta = \frac{O}{H} = \frac{A_y}{A} \Rightarrow A_x = A \sin \theta \)

Kinematics:

- **Distance vs. Displacement**
  - Distance is how far something moves and it includes the path travelled.
    - Distance is a scalar.
  - Displacement is the straight-line distance from where the object started to where it ended.
    - Displacement is a vector.
    - Displacement is the change in position of an object. \( \Delta x = x_f - x_i \)
- **Speed** = \( \frac{\text{Distance}}{\text{Time}} \), is a scalar.
- Velocity, \( \vec{v} = \frac{\Delta \vec{x}}{\Delta t} \), is a vector.
- Acceleration, \( \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \), is a vector.
  - The slope of a position vs. time graph is velocity.
  - The slope of a velocity vs. time graph is acceleration.
  - On an acceleration vs. time graph, the area between the curve & the time axis is change in velocity.
  - On a velocity vs. time graph, the area between the curve & the time axis is change in position which is also called displacement.
  - In Free Fall, \( a_y = \vec{-g} = -9.81 \frac{m}{s^2} \).
    - An object is in free fall if the only force acting on it is the force of gravity. In other words: the object is flying through the vacuum you can breathe* and not touching any other objects.

* Vacuum you can breathe = no air resistance.

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• The Uniformly Accelerated Motion Equations (UAM Equations):

| \( v_x = v_{x0} + a_x t \) | \( v_f = v_i + a \Delta t \) |
| \( x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \) | \( \Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2 \) |
| \( v_x^2 = v_{x0}^2 + 2a_x (x - x_0) \) | \( v_f^2 = v_i^2 + 2a \Delta x \) |
| \( \Delta x = \frac{1}{2} (v_f + v_i) \Delta t \) |

• The AP Physics 1 UAM Equations assume \( t = 0; \Delta t = t_f - t_i = t_f - 0 = t \)

• Projectile Motion: An object flying through the vacuum you can breathe in at least two dimensions.

<table>
<thead>
<tr>
<th>( x ) direction</th>
<th>( y ) direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_x = 0 )</td>
<td>Constant Velocity</td>
</tr>
<tr>
<td></td>
<td>( a_y = -g = -9.81 \frac{m}{s^2} )</td>
</tr>
<tr>
<td>( v_x = \frac{\Delta x}{\Delta t} )</td>
<td>Uniformly Accelerated Motion</td>
</tr>
</tbody>
</table>

\( \Delta t \) is the same in both directions because it is a scalar and has magnitude only (no direction).

- Remember to break your initial velocity into its components if it is not directly in the \( x \) direction and if the initial velocity is directly in the \( x \) direction, then the initial velocity in the \( y \) direction equals zero.

- Relative Motion is Vector Addition.
  - Draw vector diagrams.
  - Break vectors into components using SOH CAH TOA.
  - Make a right triangle.
  - Use SOH CAH TOA and the Pythagorean theorem to determine the magnitude and direction of the resultant vector.

- Center of mass.
  - Only need to know center of mass qualitatively, in other words, without numbers.
  - For the purposes of translational motion, which is essentially non-rotational motion, the whole object or system of objects can be considered to be located at its center of mass. For example, an object or group of objects in projectile motion is described by only analyzing the motion of the center of mass not each individual part of the object or system.
Flipping Physics Lecture Notes:
AP Physics 1 Review of Dynamics
https://www.flippingphysics.com/ap1-dynamics-review.html

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- Inertial Mass vs. Gravitational Mass
  - Inertial mass: the measure of an object’s inertia or a measure of its resistance to acceleration.
  - Gravitational Mass: used to determine the force of gravity or weight of an object. \( \vec{F}_g = m \vec{g} \)
  - Inertial Mass and Gravitational Mass are experimentally identical.

- Newton’s First Law: “An object at rest will remain at rest and an object in motion will remain at a constant velocity unless acted upon by a net external force.”
  - Common mistake: “an object in motion will remain in motion” is wrong. It will remain at a constant velocity which means it will have a constant speed and a constant direction.
  - Common mistake: “unless acted upon by an external force.” Do not leave out the word “net”. It is the sum of all the forces that needs to be zero for an object to remain at rest or at a constant velocity.

- Newton’s Second Law: \( \sum \vec{F} = m \vec{a} \)
  - It is arranged differently on the equation sheet: \( \vec{a} = \frac{\sum \vec{F}}{m} \), but it is the same equation.
  - When you use Newton’s Second Law, you must identify object(s) and direction.
  - Free Body Diagrams: always draw them to use Newton’s Second Law.
    - On the AP Test, do NOT break forces in to components in your initial Free Body Diagram.

- The Force of Gravity or Weight of an object is always down. \( \vec{F}_g = m \vec{g} \)

- The Force Normal is caused by a surface, is normal or perpendicular to the surface and always a push.

- Dimensions for Force are Newtons, N: \( \sum \vec{F} = m \vec{a} \Rightarrow N = \frac{kg \cdot m}{s^2} \)

- The Force of Friction is parallel to the surface, opposes motion and independent of the direction of the force applied. On equation sheet: \( |\vec{F}_{fj}| \leq \mu |\vec{F}_n| \), which works out to be three equations because we have two types of friction.
  - Static or non-moving friction: the two surfaces do not slide relative to one another.
    \( \vec{F}_{sf} \leq \mu_s \vec{F}_n \) and \( \mu_{sf} \vec{F}_n = \mu_{n} \vec{F}_n \)
  - Kinetic or moving friction: the two surfaces do slide relative to one another. \( \vec{F}_{kf} = \mu_k \vec{F}_n \)
  - For two surfaces, the coefficient of kinetic friction is always less that the coefficient of static friction. \( \mu_k < \mu_s \)

- Newton’s Third Law: \( \vec{F}_{21} = -\vec{F}_{12} \). For every force from object one on object two there is an equal but opposite force from object two on object one where both forces are vectors.

- Newton’s Third Law Force Pairs or Action-Reaction Pairs:
  - Act on two different objects and act simultaneously.
  - Inclines: Break the Force of Gravity in to its components that are parallel and perpendicular to the incline. \( F_{g\parallel} = mg \sin \theta \) & \( F_{g\perp} = mg \cos \theta \)

- Translational Equilibrium: \( \sum \vec{F} = 0 = m \vec{a} \Rightarrow \vec{a} = 0 \)
  - The object is either at rest or moving with a constant velocity.
• \[ \Delta E = W = Fd = Fd \cos \theta \] : In terms of an object or a group of objects which we call the system, the change in energy of the system equals the work done on the system which is equal to force times displacement times the angle between the force and the displacement. Work causes a change in energy of the system.
  o \( F \parallel = F \cos \theta \) : The force parallel to the displacement is the force times the cosine of the angle between the force and the displacement of the object.
  o Identify which force you are using in the work equation.
  o Use the magnitude of the force and the displacement.
  o Dimensions for Work are Joules or Newtons times meters:

• Three types of mechanical energy:
  o **Kinetic Energy**: \( KE = K = \frac{1}{2}mv^2 \) (can’t be negative)
  o Dimensions for energy are also Joules:
    \[
    KE = \frac{1}{2}mv^2 \Rightarrow (kg) \left( \frac{m}{s} \right)^2 = \frac{kg \cdot m^2}{s^2} = \left( \frac{kg \cdot m}{s^2} \right)(m) = N \cdot m = J
    \]
  o **Elastic Potential Energy**: \( PE_e = U_s = \frac{1}{2}kx^2 \) (can’t be negative)
  o **Gravitational Potential Energy**: \( PE_g = mgh \) or \( \Delta U_g = mg \Delta y \)
    - \( PE_g \) can be negative. If the object is below the horizontal zero line, then \( h \), the vertical height above the zero, is line negative.

• Work and Energy are Scalars!

• Conservation of Mechanical Energy: \( ME_i = ME_f \)
  o Valid when there is no energy converted to heat, light or sound due to friction.
  o Identify the initial and final points. Identify the horizontal zero line.
  o Substitute in mechanical energies that are present.

• If there is friction & you need to use energy: \( W_f = \Delta ME \) (Does not work when there is a force applied.)

• Power, the rate at which work is done or energy is transferred into or out of the system.
  o \[ P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \frac{Fd \cos \theta}{\Delta t} = Fv \cos \theta \]
  o Dimensions for Power are Watts which are Joules per second:
    - \( P = \frac{\Delta E}{\Delta t} \Rightarrow \frac{J}{s} = \text{watts} \ & 746 \text{watts} = 1 \text{hp} \)

• Hooke’s Law: \( |F_s| = k|x| \) : The force of a spring is linearly proportional to the displacement from equilibrium position.
  o The slope of a graph of Force of a Spring vs. displacement from equilibrium position is the spring constant. \( \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{F}{x} = k \)
  o Typical dimensions for the spring constant are Newtons per meter: \( \frac{F_s}{x} = k \Rightarrow \frac{N}{m} \)
Momentum:  \( \vec{p} = m\vec{v} \) (remember, momentum is a vector)

- Dimensions for momentum have no special name:  
  \[ \vec{p} = m\vec{v} \Rightarrow \text{kg} \cdot \frac{m}{s} \]

Conservation of momentum:  \( \sum \vec{p}_i = \sum \vec{p}_f \) (during all collisions and explosions)

- Collisions in two dimensions: 2 different equations:  
  \[ \sum \vec{p}_{xi} = \sum \vec{p}_{xf} \quad \text{&} \quad \sum \vec{p}_{yi} = \sum \vec{p}_{yf} \]

Types of collisions:

<table>
<thead>
<tr>
<th>Type of Collision</th>
<th>Is Momentum Conserved?</th>
<th>Is Kinetic Energy Conserved?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic (bounce)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Perfectly Inelastic (stick)</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

- Many collisions are in between Elastic and Perfectly Inelastic. They are called Inelastic collisions. During inelastic collisions the objects bounce off of one another, momentum is conserved however Kinetic Energy is not conserved. Elastic and Perfectly Inelastic collisions are the two ideal extremes.

Rearranging Newton’s Second Law in terms of momentum:

- \[ \sum \vec{F} = m\vec{a} = m\left( \frac{\Delta \vec{v}}{\Delta t} \right) = m\left( \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \right) = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \Delta \vec{p} \Rightarrow \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \]

- Gives us the equation for impulse:  
  \[ \Delta \vec{p} = \sum \vec{F} \Delta t = \vec{J} = \text{Impulse} \]

- The Impulse Approximation gives us the equation on the equation sheet:  
  \[ \sum \vec{F} \approx \vec{F}_{\text{impact}} \Rightarrow \Delta \vec{p} = \vec{F}_{\text{impact}} \Delta t = \vec{J} = \text{Impulse} \]

- On a Force of Impact vs. time graph, the area between the curve & the time axis is impulse.

- Dimensions for impulse:  
  \[ \Delta \vec{p} = \vec{F}_{\text{impact}} \Delta t \Rightarrow N \cdot s = \frac{\text{kg} \cdot m}{s} \]
• Angular Velocity: \( \omega = \frac{\Delta \theta}{\Delta t} \left( \frac{\text{rad}}{s} \text{ or } \frac{\text{rev}}{\text{min}} \right) \)

  o Remember for conversions: \( 1 \text{ rev} = 360^\circ = 2\pi \text{ radians} \)

• Angular Acceleration: \( \alpha = \frac{\Delta \omega}{\Delta t} \left( \frac{\text{rad}}{s^2} \right) \)

• Uniformly Angularly Accelerated Motion, UaM, is just like UAM, only it uses angular variables:
  o Equations are valid when \( \alpha = \text{constant} \)

<table>
<thead>
<tr>
<th>Uniformly Accelerated Motion, UAM</th>
<th>Uniformly Angularly Accelerated Motion, UaM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_x = v_{x0} + a_x t )</td>
<td>( \omega = \omega_0 + \alpha t )</td>
</tr>
<tr>
<td>( x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2 )</td>
<td>( \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 )</td>
</tr>
<tr>
<td>( v_x^2 = v_{x0}^2 + 2a_x (x - x_0) )</td>
<td>( \omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta )</td>
</tr>
<tr>
<td>( \Delta x = \frac{1}{2} (v_f + v_i) \Delta t )</td>
<td>( \Delta \theta = \frac{1}{2} (\omega_f + \omega_i) \Delta t )</td>
</tr>
</tbody>
</table>

• Tangential velocity is the linear velocity of an object moving along a circular path. \( \vec{v}_t = r\omega \)
  o The direction of tangential velocity is tangent to the circle and normal to the radius.
  o Tangential velocity is a linear velocity so it has the same dimensions as linear velocity: \( \frac{m}{s} \)

• Centripetal Force and Centripetal Acceleration:
  o Centripetal means “Center Seeking”
  o Centripetal force is the net force in the in direction or the “center seeking” force which causes the acceleration of the object in toward the center of the circle which is the centripetal or “center seeking” acceleration.
  o Centripetal Force, \( \sum F_{in} = m\vec{a}_c \):
    - Not a new force.
    - Never in a Free Body Diagram.
    - The direction “in” is positive and the direction “out” is negative.
  o Centripetal Acceleration, \( \vec{a}_c = \frac{v_f^2}{r} = r\omega^2 \)

• The Period, \( T \), is the time for one full cycle or revolution.
  o Dimensions for period: seconds or seconds per cycle.

• The Frequency, \( f \), is the number of cycles or revolutions per second.
  o Dimensions for frequency are cycles per second which are called Hertz, Hz: \( f \Rightarrow \frac{\text{cycles}}{\text{sec}} = \text{Hz} \)
  o Frequency and Period are inversely related: \( T = \frac{1}{f} \)
• We can use the equation for angular acceleration to derive an equation on the equation sheet:

\[ \omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi \text{ rad}}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{1}{f} \]

• The Conical Pendulum Example:

\[ \sin \theta = \frac{O}{H} = \frac{\vec{F}_T}{\vec{F}_L} \Rightarrow \vec{F}_T = \vec{F}_L \sin \theta \]

\[ \cos \theta = \frac{A}{H} = \frac{\vec{F}_T}{\vec{F}_L} \Rightarrow \vec{F}_T = \vec{F}_L \cos \theta \]

\[ \sum F_y = F_{T_r} - F_g = ma_y = m(0) \]

\[ \Rightarrow F_{T_r} = F_T \cos \theta = mg \]

\[ \sum F_{in} = F_{T_m} = \vec{F}_T \sin \theta = ma_c = m\left(\frac{v_i^2}{r}\right) \]

\[ v_i = r \omega \text{ or } v_i = \frac{\Delta x}{\Delta t} = \frac{C}{T} = \frac{2\pi r}{T} \]

We could even substitute further:

\[ F_T \cos \theta = mg \Rightarrow F_T = \frac{mg}{\cos \theta} \]

\[ \vec{F}_T \sin \theta = m\left(\frac{v_i^2}{r}\right) \Rightarrow \left(\frac{mg}{\cos \theta}\right) \sin \theta = m\left(\frac{2\pi r}{T}\right)^2 \Rightarrow g \tan \theta = \frac{4\pi^2 r^2}{T^2} = \frac{4\pi^2 r}{T^2} \]

And solve for the radius in terms of the length of the string.

\[ \sin \theta = \frac{O}{H} = \frac{r}{L} \Rightarrow r = L \sin \theta \]

\[ g \tan \theta = \frac{4\pi^2 r}{T^2} \Rightarrow \frac{g}{\cos \theta} = \frac{4\pi^2 L \sin \theta}{T^2} \Rightarrow \frac{g}{\cos \theta} = \frac{4\pi^2 L}{T^2} \Rightarrow T^2 = \frac{4\pi^2 L \cos \theta}{g} \]

And we end with an expression for the period of the circular motion.
- Torque, the ability to cause an angular acceleration of an object: \( \vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \vec{F} \sin \theta \)
  - The moment arm or lever arm is: \( \vec{r} = \vec{r} \sin \theta \)
  - A larger moment arm will cause a larger torque.
    - Maximize torque by maximizing \( r \), the distance from axis of rotation to the force.
    - Maximize torque by using an angle of 90° because \( \sin(90^\circ) = 1 \)
  - Dimensions for Torque are Newtons meters, N·m, not to be confused with Joules for energy.
  - Torque is a vector.
    - For direction use clockwise and counterclockwise. (sadly, not the right hand rule)
- Rotational form of Newton’s Second Law: \( \sum \vec{\tau} = I \vec{\alpha} \)
- Moment of Inertia or Rotational Mass:
  - For a system of particles: \( I = \sum m_i r_i^2 \)
  - Dimensions for Moment of Inertia: \( I = \sum m_i r_i^2 \Rightarrow kg \cdot m^2 \)
  - For a rigid object with shape the value or the equation will be given to you. For example:
    - \( I_{\text{solid cylinder}} = \frac{1}{2}MR^2 \); \( I_{\text{thin hoop}} = MR^2 \); \( I_{\text{solid sphere}} = \frac{2}{5}MR^2 \);
    - \( I_{\text{thin spherical shell}} = \frac{2}{3}MR^2 \); \( I_{\text{rod}} = \frac{1}{12}ML^2 \); \( I_{\text{rod about end}} = \frac{1}{3}ML^2 \)
    - With the exception of \( I_{\text{rod about end}} \), these are all about the center of mass of the object.
- Rotational Kinetic Energy: \( KE_{\text{rot}} = \frac{1}{2}I\omega^2 \)
  - \( KE_{\text{rot}} \), like translational energy, is in Joules, J.
  - Rolling without slipping: When an object rolls down a hill, it will gain not only translational kinetic energy but also rotational kinetic energy. Which means, the higher the moment of inertia, the higher the rotational kinetic energy of the object and therefore the lower amount of energy that will be left over for translational kinetic energy and therefore a lower final linear velocity.
    - Using Conservation of Mechanical Energy: \( ME_i = ME_f \Rightarrow PE_{gi} = KE_{rot f} + KE_{tf} \)
    - Also need the equation for the velocity of the center of mass of a rigid object rolling without slipping: \( v_{cm} = R\omega \)
- Angular Momentum: \( \vec{L} = I\vec{\omega} \)
  - Dimensions for Angular Momentum: \( \vec{L} = I\vec{\omega} \Rightarrow (kg \cdot m^2) \left( \frac{\text{rad}}{s} \right) = kg \cdot m^2 \)
- Angular Impulse: \( \Delta \vec{L} = \vec{\tau}_{\text{impact}} \Delta t = \text{Angular Impulse} \)
  - Dimensions for Angular Impulse: \( \Delta \vec{L} = \vec{\tau}_{\text{impact}} \Delta t \Rightarrow N \cdot m \cdot s \)
• Newton’s Universal Law of Gravitation: \( F_g = \frac{Gm_1m_2}{r^2} \)
  
  o Universal Gravitational Constant: \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \)
  
  o \( r \) is not defined as the radius, it is defined as the distance between the centers of mass of
  the two objects which can be confusing because sometimes it does work out to be the
  radius.
  
  o \( F_g = m \ddot{g} \) is planet specific.
  
  o \( F_g = \frac{Gm_1m_2}{r^2} \) is universally true.
  
  o We can combine the two to solve for the acceleration due to gravity on Earth (or any large,
    celestial body): \( F_g = m \ddot{g} = \frac{Gm_mE}{(R_E + \text{alt})^2} \Rightarrow g = \frac{Gm_E}{(R_E + \text{alt})^2} \)
  
  • The gravitational field is approximately constant on the surface of the Earth because our height is so
    small compared to the radius of the Earth: \( h_{\text{avg}} \approx 1.8 \text{ m}, R_E \approx 6,370,000 \text{ m} \)
  
  • The gravitational field is not constant from a global perspective and decreases
    as altitude increases, this can be shown using a vector field diagram.
  
  • Solving for the speed of the satellite in orbit around the Earth:

  \[ \sum F_i = F_g = ma_c = \frac{Gm_mE}{r^2} = \frac{v^2}{r} \]

  \[ \Rightarrow v_t = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{Gm_E}{(R_E + \text{alt})}} \]

  • Universal Gravitational Potential Energy: \( U_g = -\frac{Gm_1m_2}{r} \)
  
  o The equation used to find gravitational potential energy in a non-uniform gravitational field.
  
  o \( U_g \leq 0 \): The zero line is infinitely far away. \( U_g = -\frac{Gm_1m_2}{\infty} \approx 0 \)
  
  o A single object can not have Universal Gravitational Potential Energy. Universal
    Gravitational Potential Energy is defined as the Gravitational Potential Energy that exists
    between two objects.
    - Technically Gravitational Potential Energy in a constant gravitational field:
      \( PE_g = mgh \), is the gravitational potential energy that exists between the object
      and the Earth. So even \( PE_g \) requires two objects.
The mass-spring system shown at right is in simple harmonic motion. The mass moves through the following positions: 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, etc.

Simple Harmonic Motion (SHM) is caused by a Restoring Force:
- A Restoring Force is always:
  o Towards the equilibrium position.
  o Magnitude is proportional to distance from equilibrium position.

To derive the equation for position in SHM, we start by comparing simple harmonic motion to circular motion.

\[
\cos \theta = \frac{A}{H} = \frac{x}{r} \Rightarrow x = r \cos \theta \quad \text{&} \quad T = \frac{2\pi}{\omega} = \frac{1}{f} \Rightarrow \omega = 2\pi f
\]

\[
\omega = \frac{\Delta \theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\theta_f - 0}{t_f - t_i} = \frac{\theta}{t} \Rightarrow \theta = \omega t
\]

\[
x = r \cos \theta = r \cos(\omega t) = r \cos [(2\pi f)(t)] = A \cos [(2\pi f)(t)]
\]

(letting \( r = A \))

Looking at the graphs ...
The period of a mass-spring system: $T_s = 2\pi \sqrt{\frac{m}{k}}$ is independent of amplitude and acceleration due to gravity.

The period of a pendulum: $T_p = 2\pi \sqrt{\frac{L}{g}}$ is independent of amplitude and mass.
A wave is the motion of a disturbance traveling through a medium not the motion of the medium itself.

- The disturbance of the medium is energy traveling through a medium.
- Wave Pulse: A single wave traveling through a medium.
- Periodic Wave: Many wave pulses at specific, periodic time intervals.
- The energy moves through the medium as the wave pulse, however, the overall displacement of the medium is zero. $\Delta x_{\text{medium}} = 0$

Transverse wave: the disturbance of the medium is perpendicular to the direction of wave propagation. (shown at right)

- Waves in rope, Ripples on a Pond

Longitudinal wave: the disturbance of the medium is parallel to the direction of wave propagation.

- Density on $y$-axis instead of $y$-position.
- Sound in Air, Seismic waves in the Earth

Electromagnetic waves are transverse waves that do not need a medium to travel through. They are the only waves we know of that do not need a medium.

- The distance between two successive crests is called the wavelength, $\lambda$.
- The time it takes for one full cycle or for one wavelength to pass a point is called the period, $T$.

Using the equation for velocity, we can determine the equation for the velocity of a wave:

- $v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T}$ and $f = \frac{1}{T} \Rightarrow v = f \lambda$
- Simple Harmonic Motion, SHM, does not have a wavelength, so you cannot use $v = f \lambda$ with SHM.

Waves are not physical objects; they are energy traveling through a medium. Therefore, waves can occupy the same space at the same time. We determine their combined amplitude using superposition.

- Constructive Interference
  - Waves add together to create a larger amplitude.
- Total Destructive Interference
  - Waves cancel one another out to create an amplitude of zero.
Standing waves: Periodic waves are reflected and inverted and interfere with one another creating standing waves.

- Nodes: Locations of total destructive interference.
- Antinode: Locations of constructive interference.
- \( n \) is called the Harmonic Number. \( n = 1 \) is the fundamental frequency and the 1st harmonic. \( n = 2 \) is the 2nd harmonic, etc.
- Pitch is our brain’s interpretation of frequency. 440 Hz is typically concert pitch and is the A above middle C.
- Open pipe instrument (like the flute) is open at both ends and has the same equation for frequency as a stringed instrument.
- Closed pipe instrument (like the clarinet) is open on one end and closed on the other, therefore, it has a slightly different equation as derived on the next page.

Standing waves will only occur at specific wavelengths and the wavelengths are determined by the length of the string or air column in the wind instrument!

An open end of a wind instrument creates an antinode and a closed end creates a node.

Stringed Instrument and Open Pipe:

- Fundamental Frequency or 1st harmonic: \( \frac{1}{2} \lambda = L \Rightarrow \lambda = 2L \) & \( v = f \lambda \Rightarrow f = \frac{v}{\lambda} = \frac{v}{2L} = 1 \left( \frac{v}{2L} \right) \)
- 2nd harmonic: \( \lambda = L \) & \( f = \frac{v}{\lambda} = \frac{v}{L} = 2 \left( \frac{v}{2L} \right) \)
- 3rd harmonic: \( \frac{3}{2} \lambda = L \Rightarrow \lambda = \frac{2L}{3} \) & \( f = \frac{v}{\lambda} = \frac{v}{\frac{2L}{3}} = \frac{3v}{2L} = 3 \left( \frac{v}{2L} \right) \)

- \( f = n \left( \frac{v}{2L} \right); n = 1, 2, 3, \ldots \)
Closed Pipe Instrument:

- Fundamental Frequency or 1st harmonic: \( \frac{1}{4} \lambda = L \Rightarrow \lambda = 4L \) & \( f = \frac{v}{\lambda} \Rightarrow f = \frac{v}{4L} = \frac{1}{4} \left( \frac{v}{4L} \right) \)

- 3rd harmonic: \( \frac{3}{4} \lambda = L \Rightarrow \lambda = \frac{4L}{3} \) & \( f = \frac{v}{\lambda} = \frac{v}{\frac{4L}{3}} = 3 \left( \frac{v}{4L} \right) \)

- 5th harmonic: \( \frac{5}{4} \lambda = L \Rightarrow \lambda = \frac{5L}{3} \) & \( f = \frac{v}{\lambda} = \frac{v}{\frac{5L}{3}} = 5 \left( \frac{v}{4L} \right) \)

- \( f = m \left( \frac{v}{4L} \right) ; \ m = 1, 3, 5, \ldots \)

Beat Frequency: \( f_{\text{beat}} = |f_1 - f_2| \)

- When two notes are played that have frequencies that are close to one another, the constructive and destructive interference pattern creates “beats” in the sound.
- For example, \( f_{\text{beat}} = |440 - 441| = 1 \text{ hz} \), will sound with 1 “beat” per second.

Doppler Effect: The change in the wavelength and therefore frequency and therefore pitch we hear of a moving sound source. (The observer can also move to cause the same effect.) Pictures!

- As the sound source moves towards the observer the crests are closer to one another and therefore the wavelength is decreased. \( v = f \lambda \), therefore the frequency is increased and we hear a higher pitch.
- As the sound source moves away from the observer the crests are farther apart and therefore the wavelength is increased. \( v = f \lambda \), therefore the frequency is decreased and we hear a lower pitch.
Elementary Charge: The smallest charge of an isolated particle. \( e = 1.6 \times 10^{-19} \text{C} \)

- Two examples: \( q_{\text{electron}} = -e = -1.6 \times 10^{-19} \text{C} \) & \( q_{\text{proton}} = +e = +1.6 \times 10^{-19} \text{C} \)

The electron is a fundamental particle, however, the proton is not a fundamental particle.

Protons and neutrons are composed of “up” and “down” quarks: \( q_{\text{up quark}} = \frac{2}{3}e \) & \( q_{\text{down quark}} = -\frac{1}{3}e \)

- Proton is composed of 2 “up” quarks and 1 “down” quark.
  \[ q_{\text{proton}} = 2q_{\text{up quark}} + 1q_{\text{down quark}} = 2\left(\frac{2}{3}e\right) + \left(-\frac{1}{3}e\right) = +\frac{4}{3}e - \frac{1}{3}e = +e \]

- Neutron is composed of 1 “up” quark and 2 “down” quarks.
  \[ q_{\text{neutron}} = q_{\text{up quark}} + 2q_{\text{down quark}} = \left(\frac{2}{3}e\right) + 2\left(-\frac{1}{3}e\right) = +\frac{2}{3}e - \frac{2}{3}e = 0 \]

- A quark can have a charge less than the Elementary Charge because a single quark has never been isolated; quarks are always found in groups like they are in the proton and neutron.

The Law of Charges: Unlike charges attract and like charges repel. For example:

- Two positive charges repel one another & two negative charges repel one another.
- A positive and a negative charge attract one another.

The force they repel or attract one another with is determined using Coulomb’s Law: \( F_e = \frac{kq_1q_2}{r^2} \)

- This is called the Electrostatic Force. (Also sometimes called a Coulomb Force)
- Coulomb’s Constant, \( k = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2} \)
- \( q_1 \) & \( q_2 \) are the charges on the two charged particles.
- \( r \) is not the radius, it is the distance between the centers of charge of the two charges. (Sometimes \( r \) actually is the radius, however, that is not its definition.)
- Note the similarity to Newton’s Universal Law of Gravitation: \( F_g = \frac{Gm_1m_2}{r^2} \)
  - However, comparing Coulomb’s Constant to \( G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2} \) shows that Coulomb’s Constant is about \( 10^{20} \) times greater than the Gravitational Constant. In general, the electrostatic force is much, much, much greater than the gravitational force.

Conservation of Charge: In an isolated system the total charge stays constant. For example, if we start with two electrically isolated spheres, \( q_{1i} = +4C \) & \( q_{2i} = -2C \), we touch them together and pull them apart:

\[ q_i = q_{1i} + q_{2i} = +4C + (−2C) = +2C \quad \text{&} \quad q_{1f} = q_{2f} = q_i \Rightarrow q_i = q_{1f} + q_{2f} = q_i \quad \text{&} \quad q_i = 2q_i \Rightarrow q_i = \frac{q_i}{2} = +2C = +1C \]

Each sphere ends up with \( 6.24 \times 10^{18} \) excess protons on it:

\[ q_i = n_i e \Rightarrow n_i = \frac{q_i}{e} = \frac{+1C}{1.6022 \times 10^{-19} \text{C/proton}} = 6.24 \times 10^{18} \text{protons} \]
Electric Current: The rate at which charges move.
  \[ I = \frac{\Delta q}{\Delta t} \Rightarrow C = \text{Amperes, Amps, A} \]

Conventional current is the direction that positive charges "would" flow.
  Even though it is usually negative charges flowing in the negative direction.

Resistance, \( R \): A resistor restricts the flow of charges.
  \[ R = \frac{\rho \ell}{A} \]; \( \rho = \text{resistivity}; \ell = \text{length of wire}; A = \text{Cross Sectional Area} \]
  Resistivity is a material property.

Electric Potential Difference, \( \Delta V = \frac{\Delta P_{\text{electrical}}}{q} \)
  \[ \Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow R = \frac{\Delta V}{I} \Rightarrow V = \text{Ohm, } \Omega \]

Two resistors in series:
  Using Kirchhoff’s Loop Rule: \( \Delta V_{\text{loop}} = 0 \)
  \[ I_1 = I_2 \text{ & } \Delta V_{\text{loop}} = 0 = \Delta V_t - \Delta V_1 - \Delta V_2 \Rightarrow \Delta V_t = \Delta V_1 + \Delta V_2 \]
  \[ \Rightarrow I_{\text{eq}} = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow R_{\text{eq}} = R_1 + R_2 \Rightarrow R_{\text{series}} = R_1 + R_2 + R_3 + ... \]

Two resistors in parallel:
  Using Kirchhoff’s Junction Rule: \( \sum I_{\text{in}} = \sum I_{\text{out}} \)
  \[ \Delta V_{\text{loop}} = 0 = \Delta V_1 - \Delta V_1 \Rightarrow \Delta V_t = \Delta V_1 \]
  \[ \Delta V_{\text{loop}} = 0 = \Delta V_2 - \Delta V_2 \Rightarrow \Delta V_t = \Delta V_2 \]
  \[ \Rightarrow \Delta V_1 = \Delta V_1 \text{ & } \sum I_{\text{in}} = \sum I_{\text{out}} \Rightarrow I_t = I_1 + I_2 \]
  \[ \Rightarrow \frac{\Delta V_t}{R_{\text{eq}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{\text{par}} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \right)^{-1} \]
Electric Power is the rate at which electric potential energy is being converted to heat and light. Also sometimes called the rate at which energy is dissipated in the circuit element.

- \( P = I\Delta V \) (the only equation for electric power on the equation sheet)

- Example Problem: Find the power dissipated in resistor #2.

  - Resistors 1 and equivalent resistor 23 are in series:
    \[
    R_{eq} = R_1 + R_{23} = 1 + 1.2 = 2.2 \ \Omega
    \]
  
  - We can find the current through the battery, which is the same as the current through resistor 1:
    \[
    \Delta V = I R_{eq} \Rightarrow I = \frac{\Delta V}{R_{eq}} = \frac{6}{2.2} = 2.72 \ A = I_1
    \]
  
  - We can now find the electric potential difference across resistor 1:
    \[
    \Delta V_1 = I_1 R_1 = (2.72)(1) = 2.72 \ V
    \]
  
  - Now we can find the electric potential difference across equivalent resistor 23, which is the same as the electric potential difference across resistor 2:
    \[
    \Delta V = \Delta V_1 + \Delta V_{23} \Rightarrow \Delta V_{23} = \Delta V - \Delta V_1 = 6 - 2.72 = 3.28 \ V = \Delta V_2
    \]
  
  - We have what we need to find the electric power in resistor 2:
    \[
    P_2 = \frac{(\Delta V_2)^2}{R_2} = \frac{(3.28)^2}{2} = 5.3537 \approx 5.4 \ \text{watts}
    \]
A wave is the motion of a disturbance traveling through a medium not the motion of the medium itself.

- The disturbance of the medium is energy traveling through a medium.
- Wave Pulse: A single wave traveling through a medium.
- Periodic Wave: Many wave pulses at specific, periodic time intervals.
- The energy moves through the medium as the wave pulse, however, the overall displacement of the medium is zero. \( \Delta x_{\text{medium}} = 0 \)

Transverse wave: the disturbance of the medium is perpendicular to the direction of wave propagation. (shown at right)
- Waves in rope, ripples on a pond

Longitudinal wave: the disturbance of the medium is parallel to the direction of wave propagation.
- Density on y-axis instead of y-position.
- Sound in air, seismic waves in the Earth

Electromagnetic waves are transverse waves that do not need a medium to travel through. They are the only waves we know of that do not need a medium.

- The distance between two successive crests is called the wavelength, \( \lambda \).
- The time it takes for one full cycle or for one wavelength to pass a point is called the period, \( T \).

Using the equation for velocity, we can determine the equation for the velocity of a wave:
- \( v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} \) \& \( f = \frac{1}{T} \Rightarrow v = f \lambda \)
- Simple Harmonic Motion, SHM, does not have a wavelength, so you cannot use \( v = f \lambda \) with SHM.

Waves are not physical objects; they are energy traveling through a medium. Therefore, waves can occupy the same space at the same time. We determine their combined amplitude using superposition.

- Constructive Interference
  o Waves add together to create a larger amplitude.
- Total Destructive Interference
  o Waves cancel one another out to create an amplitude of zero.
Standing waves: Periodic waves are reflected and inverted and interfere with one another creating standing waves.

- Nodes: Locations of total destructive interference.
- Antinode: Locations of constructive interference.

- $n$ is called the Harmonic Number. $n = 1$ is the fundamental frequency and the 1st harmonic. $n = 2$ is the 2nd harmonic, etc.
- Pitch is our brain’s interpretation of frequency. 440 Hz is typically concert pitch and is the A above middle C.
- Open pipe instrument (like the flute) is open at both ends and has the same equation for frequency as a stringed instrument.
- Closed pipe instrument (like the clarinet) is open on one end and closed on the other, therefore, it has a slightly different equation as derived on the next page.

Standing waves will only occur at specific wavelengths and the wavelengths are determined by the length of the string or air column in the wind instrument!

An open end of a wind instruments creates an antinode and a closed end creates a node.

Stringed Instrument and Open Pipe:

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- 2nd harmonic: $\lambda = L \quad & \quad f = \frac{v}{\lambda} = \frac{v}{L} = 2 \left( \frac{v}{2L} \right)$

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- $f = n \left( \frac{v}{2L} \right); \quad n = 1, 2, 3, ...$
Closed Pipe Instrument:

- Fundamental Frequency or 1st harmonic: $\frac{1}{4} \lambda = L \Rightarrow \lambda = 4L$ & $v = f \lambda \Rightarrow f = \frac{v}{\lambda} = \frac{V}{4L} = 1\left(\frac{V}{4L}\right)$
  
- 3rd harmonic: $\frac{3}{4} \lambda = L \Rightarrow \lambda = \frac{4L}{3}$ & $f = \frac{V}{\frac{4L}{3}} = 3\left(\frac{V}{4L}\right)$
  
- 5th harmonic: $\frac{5}{4} \lambda = L \Rightarrow \lambda = \frac{5L}{3}$ & $f = \frac{V}{\frac{5L}{3}} = 5\left(\frac{V}{4L}\right)$

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- As the sound source moves away from the observer the crests are farther apart and therefore the wavelength is increased. $v = f \lambda$, therefore the frequency is decreased and we hear a lower pitch.
Let me be clear about what I mean by “memorize”: I mean you should have the equation memorized, know what it means and know when you can use it. This is a lot more than just being able to write down the equation.

The following equations are not on the Equation Sheet provided by the AP College Board for the AP Physics 1 exam:

- **speed** = \( \frac{\text{distance}}{\text{time}} \); \( \vec{v} = \frac{\Delta \vec{x}}{\Delta t} \); \( \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \)
  - Please make sure you understand the differences between vectors and scalars, please.*

- \( \Delta x = \frac{1}{2} (v_i + v_f) \Delta t \)
  - This is another Uniformly Accelerated Motion (UAM) equation you should know.

- \( F_{\parallel} = mg \sin \theta \) & \( F_{\perp} = mg \cos \theta \)
  - When an object is on an incline, we often need to sum the forces in the parallel and perpendicular directions, which necessitates resolving the force of gravity into its components in the parallel and perpendicular directions.
  - Note: theta in this equation is the incline angle.

- Equations having to do with Mechanical Energy:
  - \( ME_i = ME_f \) : Conservation of Mechanical Energy can be used when there is no work done by the force of friction or the force applied.
  - \( W_f = \Delta ME \) : Can be used when there is no work done by the force applied.
  - \( W_{\text{net}} = \Delta KE \) : Is always true.

- \( P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \frac{Fd \cos \theta}{\Delta t} = Fv \cos \theta \)
  - This is useful because you have power in terms of velocity.

- \( \sum \vec{p}_i = \sum \vec{p}_f \)
  - Conservation of linear momentum is valid when the net force acting on the system is zero, which is true during all collisions and explosions.

- \( \ddot{\omega} = \frac{\Delta \theta}{\Delta t} \) & \( \ddot{\omega} = \frac{\Delta \ddot{\omega}}{\Delta t} \)
  - Angular velocity and angular acceleration were, sadly, left off the equation sheet.

* Yes, that’s a double please.
• \( \omega_f^2 = \omega_i^2 + 2\alpha\Delta \theta \) & \( \Delta \theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t \)
  - These two Uniformly Angularly Accelerated Motion (UαM) equations were also, sadly, left off the equation sheet.

• \( \vec{v}_t = r\vec{\omega} \)
  - The tangential velocity of an object.

• \( v_{cm} = R\omega \)
  - The velocity of the center of mass of an object rolling without slipping.

• \( \sum \vec{F}_{in} = m\vec{a}_c \)
  - The equation for the centripetal force acting on an object to keep it moving in a circle.

• \( I = \sum m_ir_i^2 \)
  - The moment of inertia or "rotational mass" of a system of particles.

• \( \sum \vec{L}_i = \sum \vec{\tau}_i \)
  - Conservation of Angular Momentum, valid when the net external torque acting on the system is zero. \( \sum \vec{\tau}_{external} = 0 \)

• \( f_{\text{beat}} = \left| f_1 - f_2 \right| \)
  - The beat frequency heard due to the interference of two similar single frequency sounds.

• \( q = ne \)
  - The net change on an object equals the number of excess charges times the elementary charge.

• \( \Delta V = \frac{\Delta \text{PE}_{electrical}}{q} \)
  - The electric potential difference equals the change in electrical potential energy divided by charge.

• \( P = I\Delta V \) is the only equation for electric power on the equation sheet.
  - However, using \( \Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow R = \frac{\Delta V}{I} \) we can find two more.

  o \( P = I\Delta V = IIR = I^2R = \left( \frac{\Delta V}{R} \right)^2 R = \frac{\Delta V^2}{R} \)
  - (This is what you should memorize for electric power.)